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## The simulation between kinematic properties of crank linkage of engine in vehicle by lagrange formula

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### Abstract

The torque will be in proportion to the crank ratio in crank and linkage mechanism. That may be fitted to the stroke ratio  $\lambda$  well. The bigger the  $\lambda$  is the bigger the torque is. The crank acceleration and piston angle will be low when the  $\lambda$  becomes big. The linkage acceleration and speed may incline as the stroke ratio and rotation is big. The movement will be big when the stroke ratio is small and the rotation is inclining on the crank linkage mechanism in vehicle.

**Keywords:** property; simulation; crank; crank linkage; stroke ratio; vehicle; engine; kinematic

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### Introduction

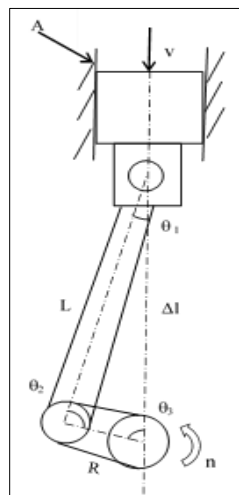
The crank linkage in engine of vehicle is important mechanism so the simulation may be done to search its properties and their effective value further and details. Because its important role in engine the optimum parameters have been solved to find the intrinsic relations among them is very significant currently [1-5]. However up to now the Kinematic equation of Lagrange formula has been a little status so it is necessary to establish the simulation to analyze the Kinematics of it. Although some searched the simulation to multibody system it is confined at three ordinations profile. They didn't search more detail to every parameter so in this study the role of parameters with movement, linkage angle and acceleration has been investigated with Lagrange formula and try to find the intrinsic relationship among them.

In this study the parameters including properties of engine crank linkage have been investigated in detail with Lagrange formula

in multibody system. It is found how the acceleration and angular speed play a role and with their other parameters link. Those properties may be clarified through drawing in order to compare with the torque. It is hoped that the effect factor which affect the torque further through comparing their value with torque.

In short the torque may be inclined has been our destination and other parameters can promote the torque through regulating the parameter value. We try to find that the parameter deeds to improve the general torque value. The stroke ratio  $\lambda$  is an important parameter in crank linkage mechanism which may affect the engine properties big so it is noticed that the reasonable crank length  $R$  and linkage  $L$  has been chosen for good combinative properties in engine of vehicle.

### Simulation of kinematic properties with Lagrange formula



**Fig 1:** the kinematic of crank linkage linkage length in the engine of vehicle.

According to Figure 1 which is kinematic graphs on the crank linkage in engine in vehicle. It is supposes that crank

$R=45\text{mm}\sim 60\text{mm}$ , crank linkage  $L=190\text{mm}\sim 210\text{mm}$ . This is the engine driving crank linkage.

A is sliding piston and cylinder wall;  $\theta_1, \theta_2$  and  $\theta_3$  is sliding piston, crank linkage and crank angle respectively;  $v$  is its speed;  $n$  is shaft rotation. The Lagrange equation is

$$\frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} = F_i, \quad (i=1,2,\dots,n) \tag{1}$$

Here  $E_k$  is kinetic of system;  $E_p$  is potential energy of system;  $q_i$  is generalized coordinate, it is a group of independent parameters that can define mechanical system movement;  $F_i$  is generalized force,  $q_i$  is a angular displacement or linear displacement;  $n$  is system generalized coordinate.

According to Lagrange Formula it has

$$F_i = m_3 v_3 + I_2 \dot{\omega}_2 + I_2 \dot{\omega}_3 + I_1 \dot{\omega}_1 - I_2 \dot{\omega}_3 - m_3 v_3 - I_2 \dot{\omega}_2 - I_2 \dot{\omega}_3 + I_1 \dot{\omega}_1 + m_3 g - I_2 \dot{\omega}_1 (1 + \dot{\omega}_2) \tag{2}$$

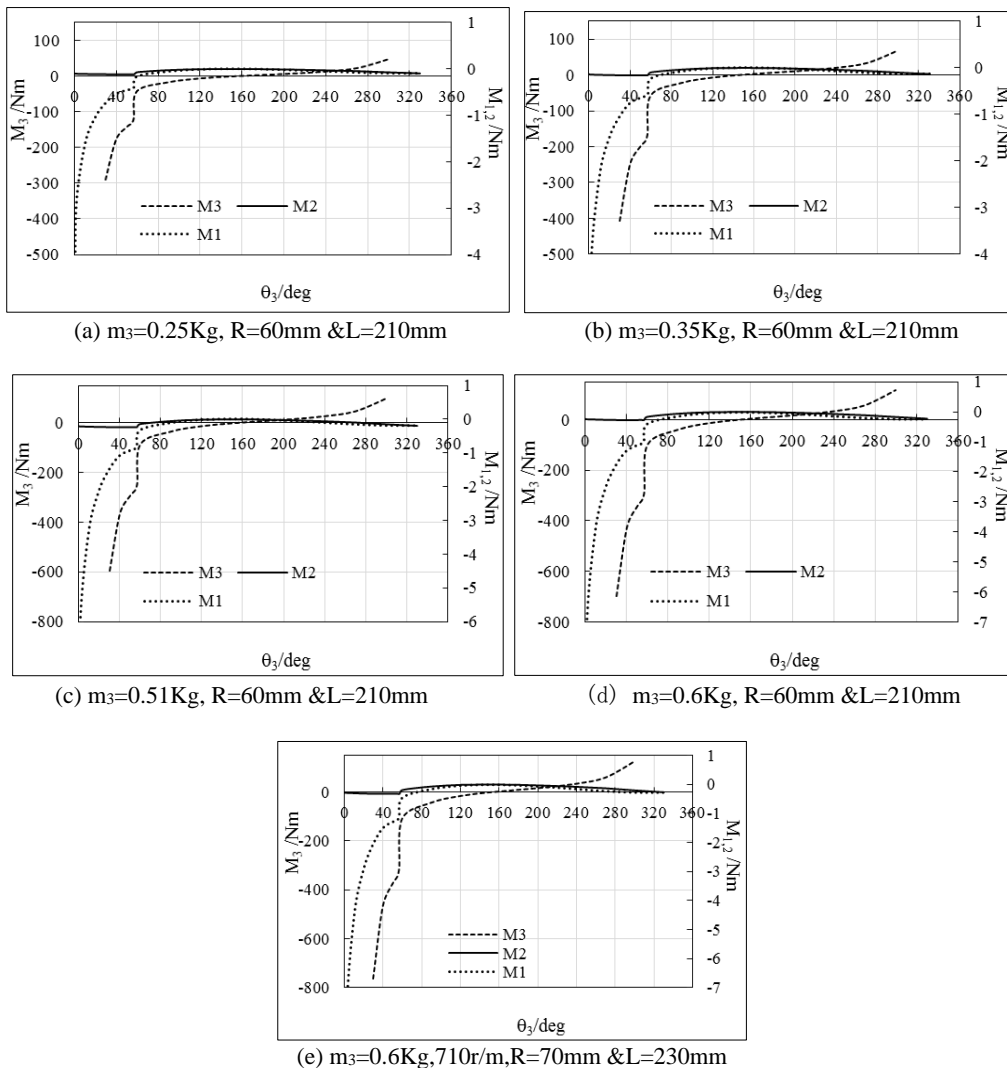
This is the dynamic equation for crank linkage of multibody system.  $\omega_1, \omega_2$  and  $\omega_3$  is sliding piston, crank linkage and crank angular angle respectively. In crank linkage of multibody system it has

$$F = \sum_{k=1}^m \left[ F_k \frac{\partial \delta s_k}{\partial q_1} \cos \alpha_k \right] + \sum_{j=1}^n \left[ M_j \frac{\partial \phi_j}{\partial q_1} \right] \tag{3}$$

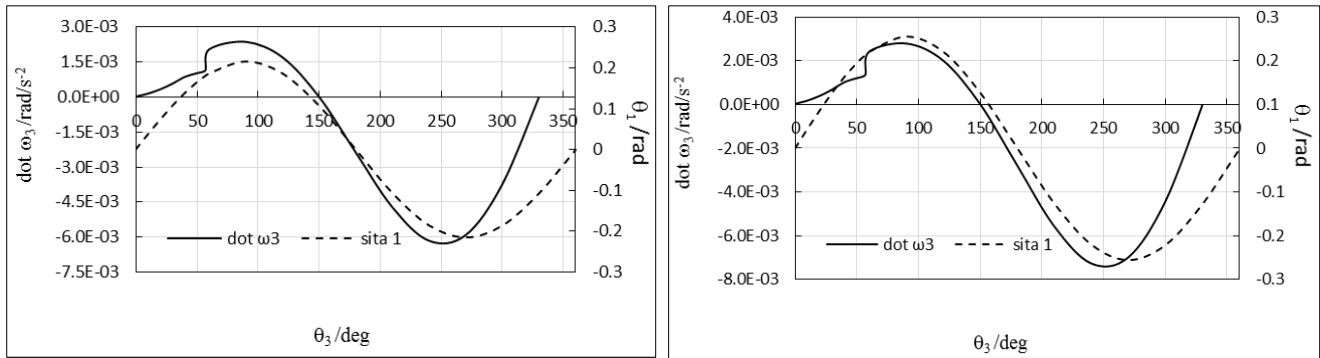
This is generalized force equation. Here  $q$  is generalized coordinate;  $\dot{q}$  is velocity of  $q$ .

**Discussions**

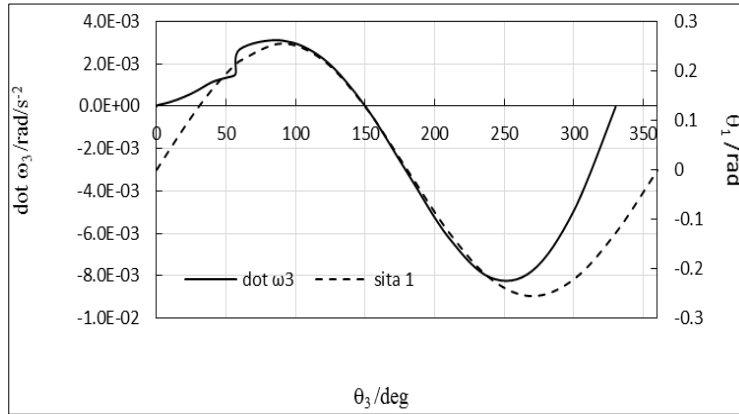
The torque will be in proportion to the piston mass, crank and linkage length in Figure 2(a~e). The torque may increase from 300Nm to 700Nm with crank mass inclining from 0.25Kg to 0.6Kg as seen in Figure 2 (a-d). Meantime the troque of crank linkage increases from 3Nm to 7Nm with the the same conditions which may be little whilst the torque of piston is the least with 0.2Nm which can be neglected. That may be fitted to the stroke ratio  $\lambda$  well. The bigger the  $\lambda$  is the bigger the torque is as seen in Figure 2(e). The crank acceleration and piston angle will be low when the  $\lambda$  becomes big as seen in Figure 3(a-c). The acceleration will increase and the angle has little change when the rotation decreases.



**Fig 2:** The relationship between torque and crank angle 3 in crank linkage with the mass of 0.25~0.6Kg, the rotation 710r/m and R & L on engine.



(a) 790r/m, R=45mm & L=210mm (b) 790r/m, R=53mm & L=210mm

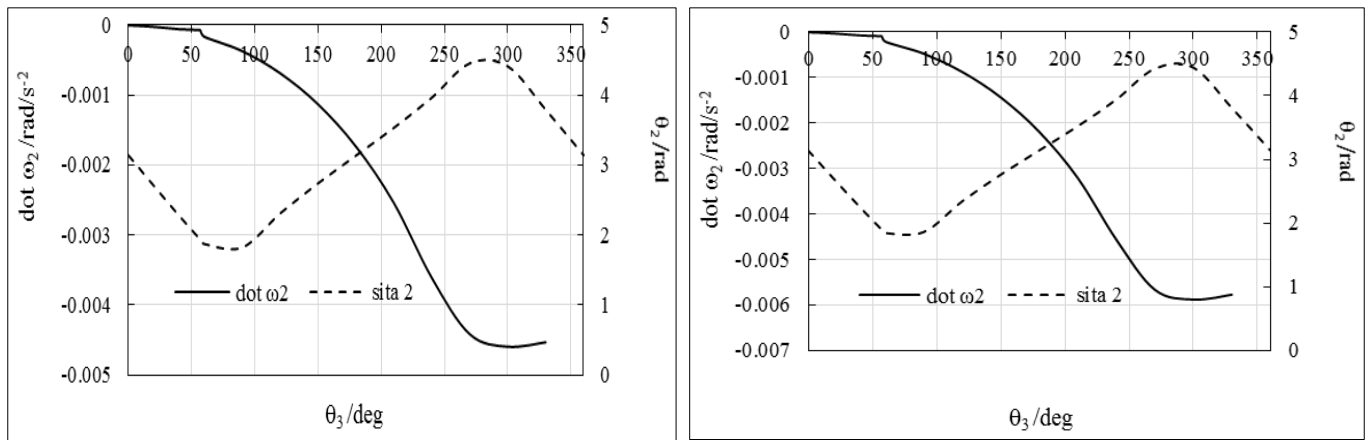


(c) 710r/m, R=53mm & L=210mm

**Fig 3:** The relationship between angular acceleration  $\dot{\omega}_3$  and crank angle  $\theta_3$  in crank linkage with time in the rotation and R & L on engine.

It is supposed that crank shaft speed is 690r/m, 710r/m, 790r/m respectively in this study. After Lagrange transformation the force and torque may be solved it is concluded as above. Other

detail parameters relation has been drawn as below in this multibody system of engine.



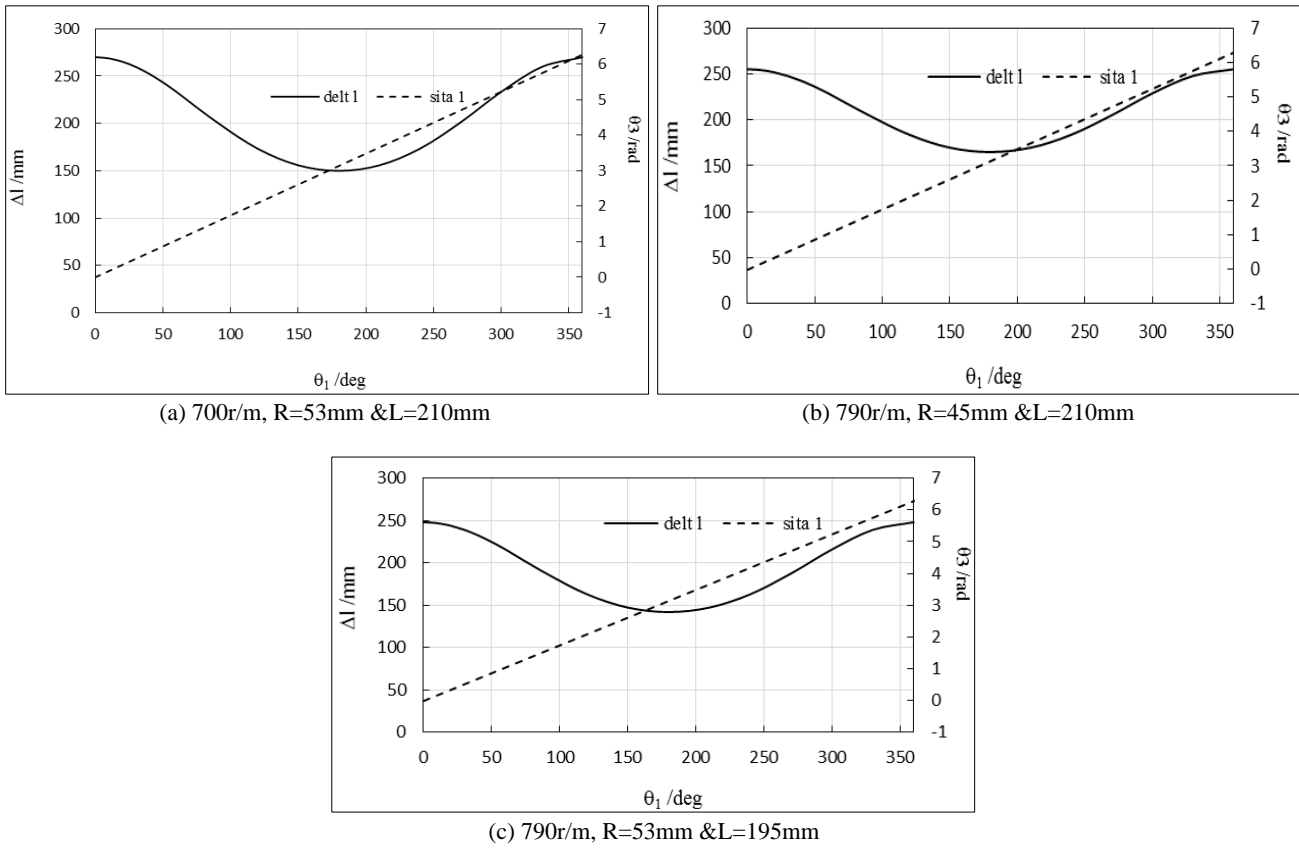
(a) 790r/m, R=53mm & L=210mm

(b) 700r/m, R=60mm & L=210mm

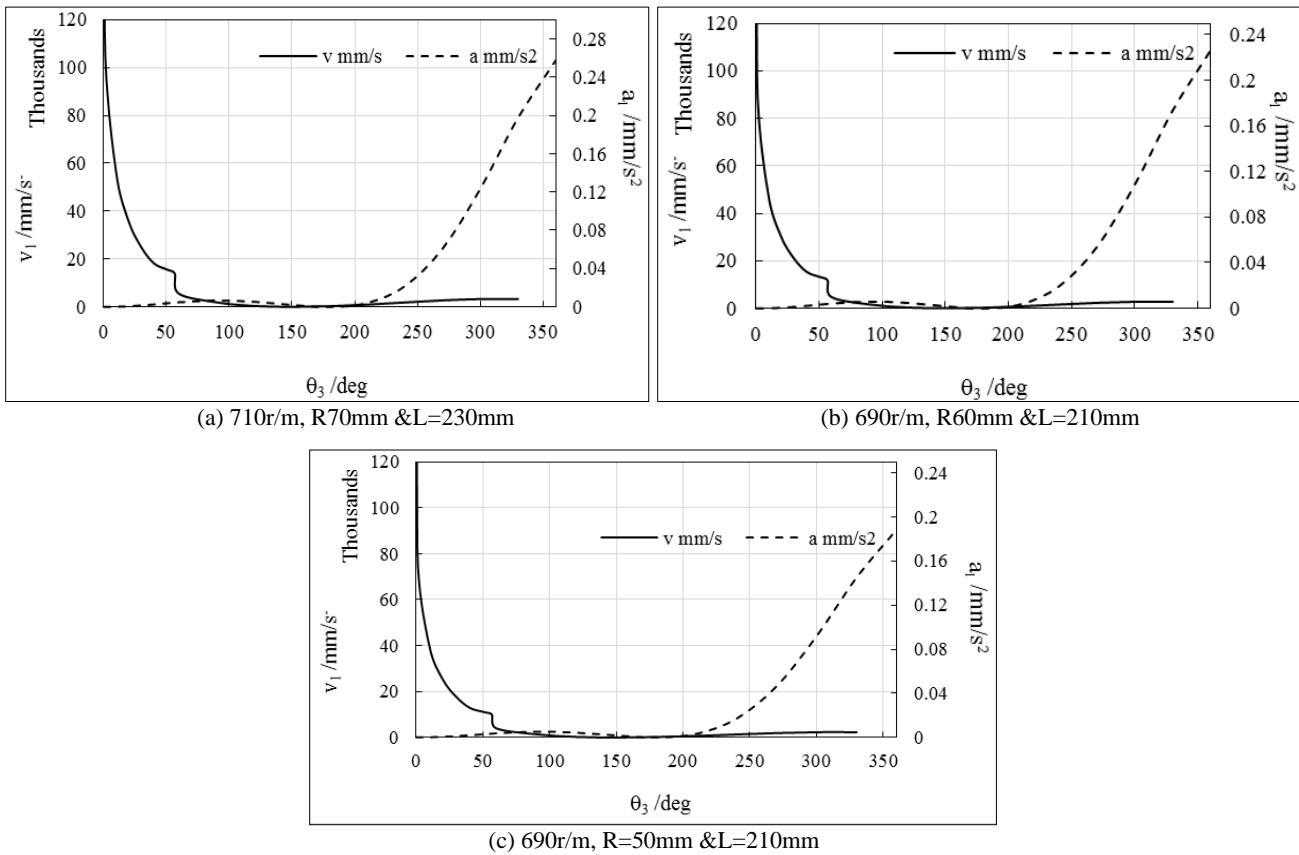
**Fig 4:** The relationship between acceleration and linkage angle in crank linkage with time in the rotation and R & L on engine.

In Figure 4 the angular acceleration will be big when the stroke ratio is big and the rotation is declining on the crank linkage mechanism in vehicle from Figure 4(a-b). In Figure 4(a-b) the dot

$\omega_2$  increases from -0.0045 to -0.006rad/s<sup>2</sup> whilst the linkage has little change when crank angle increases under the conditions of  $\lambda$  increasing and rotation decreasing.



**Fig 5:** The relationship between distance and linkage angle 3 in crank linkage with time, the rotation and R & L on engine.



**Fig 6:** The relationship between speed and acceleration in crank linkage with crank angle, rotation and R & L.

It is known that the movement will be big when the stroke ratio is big and the rotation is inclining on the crank linkage mechanism in vehicle from Figure 5(a-c) whilst the crank angle has little change. As seen in Figure 6(a-c) the piston speed and its acceleration may decline from 100m/s to 70m/s and from  $0.26\text{mm/s}^2$  to  $0.185\text{mm/s}^2$  respectively when the stroke ratio and rotation declines. Here the acceleration is too small to be neglected. In order to gain the better speed including acceleration of piston the high stroke ratio like bigger than  $0.304=70/230$  and high rotation like bigger than 710r/m in optimum design according to Figure 6(a).

### Conclusions

The torque will be big as the ratio of crank length and linkage is high. The crank acceleration and piston angle will be big as the ratio is small. These are the conclusions searching in this paper. The angular acceleration will be big when the stroke ratio is big and the rotation is inclining on the crank linkage mechanism in vehicle. The movement will be big when the stroke ratio is small and the rotation is inclining on the crank linkage mechanism.

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