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## **Cost-time trade-off in bi-criteria multi-index bulk transportation problem: A numerical approach**

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### **Abstract**

In this study, a bi-criteria multi-index bulk transportation problem is explored. The bulk transportation problem refers to a logistics scenario where the demand of each destination must be met exclusively by one source, although a single source can cater to multiple destinations. This problem balances two critical criteria: cost and time, focusing on achieving an optimal trade-off between them. A numerical example is used to illustrate the methodology for deriving the cost-time trade-off pairs in the multi-index bulk transportation problem. The least-cost solution is identified first, along with its corresponding time. Once this optimal solution is obtained, subsequent efficient trade-off pairs are determined. These trade-offs reveal how small compromises in cost can potentially reduce transportation time or vice-versa. The approach emphasizes achieving efficient transportation planning by minimizing costs while ensuring reasonable delivery times. This type of problem has real-world applications in logistics and supply chain management, where it is critical to optimize operations by balancing multiple objectives. The study provides practical insights into multi-index systems, showing how systematic methods can yield a sequence of cost-effective and time-efficient solutions to support better decision-making.

**Keywords:** Bi-criteria, transportation problem, cost time trade off, multi index, decision making

### **Introduction**

Mental health disorders are a growing public health concern in the United States. According to the National Institute of Mental Health (NIMH), over 21 million adults in the U.S. experienced a Transportation Problem is a special class of LPP where requirements are to be fulfilled by availabilities, by transporting from sources to destinations.

At first, concept of Transportation Problem, with objective of cost minimizing of Transportation has been developed by Hitchcock <sup>[1]</sup>. Then single objective Transportation Problem are discussed by several authors, named as, Balas <sup>[2]</sup>, Bhatia *et al.* <sup>[3]</sup>, Ganfinkel and Rao <sup>[4]</sup>, Hammer <sup>[5]</sup>, Seshan and Tikekar <sup>[6]</sup> and Sharma and Swarup <sup>[7]</sup>, Hakim <sup>[8]</sup>, Ahmed *et al.* <sup>[9]</sup> and Pramila and Uthra <sup>[10]</sup> using different approaches.

Bulk Transportation Problem has been taken into consideration. As Bulk Transportation Problem is something different type of Transportation Problem with some constraint that requirement is to be met from one source only. Maio and Roveda <sup>[11]</sup> introduced Bulk Transportation Problem in literature at first with the objective of minimising the BTP. Then Srinivasan and Thompson <sup>[12]</sup> studied the BTP with branch and bound algorithm further. Later on Bhatia <sup>[13]</sup> discussed a note on the time minimising BTP.

When more than one type of products are supplied through different mode of Transportation Problem, thus Multi-Index Transportation Problem is considered, which was first introduced by Schell <sup>[14]</sup> and Galler and Dwyer <sup>[15]</sup> in literature. Then Haley <sup>[16]</sup>, Junginer <sup>[17]</sup>, Pandian and Anuradha <sup>[18]</sup> and Rautman *et al.* <sup>[19]</sup> explained MITP through different approaches. Purusothan and Murthy <sup>[20]</sup> brought the new algorithm to minimise cost of MIBTP i.e. Lexi-search algorithm. Then, Bi-criteria TP is introduced where TP includes of both cost and time as objectives. Bhatia <sup>[21]</sup>, Glickman and Berger <sup>[22]</sup>, Aneja and Nair <sup>[23]</sup> and Prakash *et al.* <sup>[24]</sup> studied Bi- criteria transportation problem and bi- criteria bulk transportation problem has been studied by Prakash and Ram <sup>[25]</sup>, Prakash *et al.* <sup>[26]</sup>. Then BCMIBTP is not studied so far in literature.

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Here, the efficient solution of cost-time will be yielded by solving the numerical problem.

### Formulation

Let there be  $m$  sources,  $n$  destinations and 1 facility. The formulation of Bi-criteria multi-index bulk transportation problem is as follows:

$$\begin{aligned} &\text{Min} \left( \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} y_{ijk} : y_{ijk} = 1, \max \{t_{ijk} : y_{ijk} = 1\} \right) \\ &\text{Subject to the constraints} \\ &\sum_{j=1}^n \sum_{k=1}^l b_j y_{ijk} \leq a_i \\ &\sum_{i=1}^m \sum_{k=1}^l y_{ijk} = 1 \\ &y_{ijk} = 1 \text{ or } 0 \end{aligned}$$

Where,

$c_{ijk}$  is bulk transportation cost from  $i_{th}$  source to  $j_{th}$  destination with facility  $k$ .

$y_{ijk}$  is decision variable having value 0 or 1, value depends on whether the requirement of  $j_{th}$  destination is met or not from the source 'i' with facility 'k'.

$t_{ijk}$  is the bulk transportation time from  $i_{th}$  source to  $j_{th}$  destination with facility 'k'.

$a_i$  is the number of unit of product available at  $i_{th}$  source.

$b_j$  is the number of units required at  $j_{th}$  destination.

Let  $C_1$  and  $T_1$  be the minimum cost with corresponding time of BCMIBTP. Let  $Y_1$  be the solution that gives efficient cost-time trade-off pair  $[C_1, T_1]$ , and let  $C_2$  and  $T_2$  be another cost and time pair, this must satisfy  $C_2 (>C_1)$  and  $T_2 (<T_1)$ . Then, the solution  $Y_2$  giving solution  $[C_2, T_2]$  is said to be subsequent solution if there exist no other solution  $Y$  providing  $[C, T]$  cost-time trade-off pair s.t.  $C_1 < C < C_2$  and  $T_2 < T < T_1$ . Thus, we will get next efficient pair as  $[C_2, T_2]$  provided by  $Y_2$ . Proceeding the same way, the next efficient cost-time trade-off pair will be determined.

### Proposed Algorithm

1. Cross off the cells  $(i,j)$  from the table for which availability  $a_i$  is less than the requirement  $b_j$ .
2. Determine the minimum cost for each mode of Bulk Transportation.
3. Select minimum cost of problem  $P_1$ , let  $C_{11}$  and  $T_{11}$  be the cost and time and  $Y_{11}$  be the solution providing  $C_{11}$  and  $T_{11}$  with facility  $k=1$  for Bulk Transportation Problem and  $Y_{12}$  be the solution providing minimum cost  $C_{12}$  with corresponding time  $T_{12}$  for problem  $P_2$ .
4. Select minimum of cost associated with decision variable for each destination through facility  $k=1$  and  $k=2$ . Let  $Y_1$  be the solution obtained and let the cost and time  $C_1$  and  $T_1$ . Let  $(C_1, T_1)$  be the first efficient time-cost trade-off pair of BCMIBTP.
5. To obtain the next pair, delete the cells whose time is greater than or equal to  $T_1$  from  $P_1$  and  $P_2$  respectively. Repeat steps 2 to 3 for reduced problem. Thus  $(C_2, T_2)$  be the next efficient pair will be determined.
6. Repeat above steps until the reduced problem becomes feasible.

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