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Thermal stress prediction and start-up schedule optimization by steam turbine model combined with regenerative feed-water heating system calculation theory

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Abstract
Reducing the start-up period with a limitation of the turbine rotor stress has connections with the economical efficiency of equipments and the rotor life, and is a highly nonlinear problem which has multiple local optima. An off-design calculation model was proposed including heat calculation and thermal system theory, based on the model a specific program was developed with VC. Various cold start-up schedules were composed according to the operation regulation of 210MW steam turbine. For each schedule, steam parameters on the main parts of rotor were calculated by the program. They were the reference data for calculating of temperature and stress field for rotor with FEM (finite element method). The results of FEM were used as sample data for SVM (support vector machines). SVM was employed to build the regression model for the start-up schedule and to determine the stresses at the dangerous points. The optimal start-up parameters could be obtained by GA (genetic algorithm). The results show that the fatigue lifetime was increased by 20%, due to the 1.4% reduction of maximum Von-Mises stress, and the start-up period of the optimized schedule was reduced by 124min compared with the original schedule. The optimized start-up schedule was verified by FEM, and the maximum Von Mises stresses at the dangerous points satisfy the requirement.

Keywords: Steam turbine rotor, support vector machine, genetic algorithm (GA), start-up scheduling

Introduction

Steam turbine rotor is a main element converting the thermal energy into the mechanical energy, and its lifetime affects the safety of the whole unit. The units in power plant have their constraints for operating, such as the thermal stress related with the start-up time [1].

Especially, due to the heavy variation of the opeartion environments, some features of rotor get worse, such as thermal stress, apparent expansion, and thermal strain. If they exceed the limitation, the corresponding elements get damaged, or even destroyed causing fatal accidents.

To start-up the unit rapidly according to the requirement of serving for the peak load, it is necessary to find out the appropriate start-up curve of steam turbine, and operate the unit according to it, to improve the safety and the economical efficiency.

By now, optimization of turbine start-up in power plant has been researched in many aspects. To change the parameters of power plant under the operation in order to exam the rotor is uneconomical or even impossible. So many of models are widely used in terms of software, which can simulate the operation features of turbine, such as Modelica and MATLAB/Simulink.

F. Belkhir *et al.* [2] proposed a procedure for modeling the operation of thermal power plant based on the mass equilibrium in boiler and the energy balance in drum, and optimized its start-up scheduling with Modelica and Python. A. Tica etc. [3, 4] also built a model of a CCPP (Combined Cycle Power Plant) with Modelica, and minimized the start-up time and fuel consumption by H-MPC (Hierachical Model Predictive Control), to reduce the calculation time. They proposed a model for dynamic simulation of thermal power plant with Dymola based on Modelica, combined with the thermodynamic analysis, governing structure and stress estimation to minimize the start-up time causing the reduction of cost.

Masakazu *et al.* [5] researched the intelligent optimization system for the start-up schedule of gas turbine.

Here, velocity rise rate, durations of low revolution, high load, and initial load, respectively, are individuals, and multi-objective optimization was empolyed to minimize the start-up time, fuel consumptions and thermal stress. In some cases, thermal stress and emission of NOx are chosen as constraints. For this purpose, dynamic model of power plant was build by MATLAB^{TM}/Simulink and optimum solution was found using GRGA (Genetic Range Genetic Algorithm). Carolina V. Ponce *et al.* [6] composed a simulator which is based on the phenomenological equations about the CCPP and solar power plant, with MATLAB/ Simulink for design, estimation and control system test of ISCC (Integrated Solar Combined Cycle).

A steam turbine dynamic models for full scope power plant simulators were composed and compared with the actual operation data [7].

Based on the analysis of rotor system, speed governer system and synchronous generator, a characteristic curve of turbine-generator system was drawn under the condition of load variation, turbine-generator model was composed with MATLAB/ Simulink [8].

J. Bausa et.al considered start-up schedule as a dynamic process including the first-order ordinary differential equations (ODEs) and algebraic equation, discretize it over the time to transform it into a large scale non-linear parameter optimization problem which has over 20 000 optimization parameters and constraints [9]. Here, the CCPP was expressed as a mathematical model composed with 18 the first-order ordinary differential equations and 67 algebraic euqations.

Jose Martin *et al.* [10] presented a thermodynamic model of back pressure steam turbine with multiple extractions to determine the pressure and steam flow by extending the single extraction model.

Yong-xing Lv et.al [11] estimated the economic efficiencies of constant pressure operation and sliding pressure operation for the 600MW supercritical thermal power plant, explained the advantage of sliding pressure operation.

Green's Function Techinque (GFT) was used for the on-line stress calculation [12]. With GFT, the thermal stress could be directly determined from the parameters such as temperature, pressure and steam flow. The first step is to determine the temperatures on boundarys by the integral of thermal model according to the time, and the second is to calculate the thermal stress by GFT with the data gotten on the first step. According to the European Standard, Gerard Kosman *et al.* considered the effect of load variation such as start-up, fluctuation, optimized the start-up schedule [13]. Control variables were the live steam temperature rise rate and turbine output rise ratio, the object function was to minimize the maximum stress, start-up time was the constraint. Through the Low-Cycle Fatigue (LCF) test was used to determine the relationship between cycle stress and strain of 30Cr2MoV which is widely used for high temperature [14]. This relation could be used for design and strength calculation [14].

To estimate the fatigue properites of materials, 9 models were analysed in detail, a method was proposed to be used for estimating fatigue life of steam turbine rotor material by considerting the 400~600℃ environment [15].

The result of on-line monitoring about the turbine rotor damage during the cold start-up schedule was obtained by the transfer function gotten by the FEA [16]. The sensitivity of transfer function was estimated by Niquist curve. Mariusz deterimineded the parameters of govern stage during the start-up process based on the characteristic curve, and heat transfer coefficient with them, thermal stress on the blade groove was estimated with GFT. The result was compared with the one of FEM [17].

A method to estimate the life time was suggested by considering the technical state of turbine material on the aspect of safty and reliability [18]. To check the technical state, turbine rotor, transition area, blades of last stage or working at the phase transition region, emergency and control valves, connections of flanges, diaphragms and bolts were defined as checking points, the maximum allowable operation period was determined by 9 parameters such as creep strain, hardness, operation period, bainite spheroidized degree, friction between rotor and stator, size of crack on the disk and shaft, rotor test and calculation of lifetime.

Dong-Mei Ji et.al [19] set 12 hours as limit of start-up time, and built the cold start-up optimization model of which decision variables are the temperature rise rates on an interval of 1 hours. Sample data were composed by responses of Von-Mises stress according to the various temperature rise rates with FEM, based on them, optimum schedule was adopted of which start-up time is shorter than the given schedule as much as 150min, with SVM and PSO.

Most of models mentioned above, were built to be used for real-time simulation, from operation database and various equations reflecting the features of turbine elements. In order to simplify the calculation, steam turbine was investigated in macroscopic view. In those models, the main element of calculation was stage group in order to determine the thermal stress of turbine rotor during the start-up or shut down period more correctly, not the inlet and outlet parameters of stage groups, but of each stage are needed so as to define the temperature field of steam. And it is necessary to determine the parameters at through the concrete calculation of such paramters in a stage as pressure and enthalpy at nozzle and blade, and relative internal efficiency.

In this paper, a model was proposed to determine the parameters mentioned above at such parts as outlets of nozzle and blade with combination of the off-design regime calculation theory and thermal system theory. Based on the model, a special program was composed with VC, to calculate the steam parameters on the parts of all stages according to the inlet parameters' variation of 210MW subcritical condensing steam turbine at a power plant. According to the operation regulation of the turbine, various cold start-up schedules were composed. The FEM was employed to study the temperature field and stress field under the different schedules. Taking the calculation results as sample data, a regression model was established by SVM to determine the stress at risk point of rotor. Based on the regression model, GA was used to obtain the optimized cold start-up schedule. For GA, the temeprature rise rates and the durations of each segment in the start-up schedule, were used as control variables. According to the optimal set of temperature rise rates and durations, maximum stress of turbine rotor was calculated by SolidWorks Simulation. Based on the maximum stress of rotor, the fatigue lifetime under the optimized start-up schedule was compared with the one of the original schedule.

The Mathematical Models

The Mathematical Model of Start-up Optimization

Figure 1 shows the main thermal system dawing of the 210MW turbine.

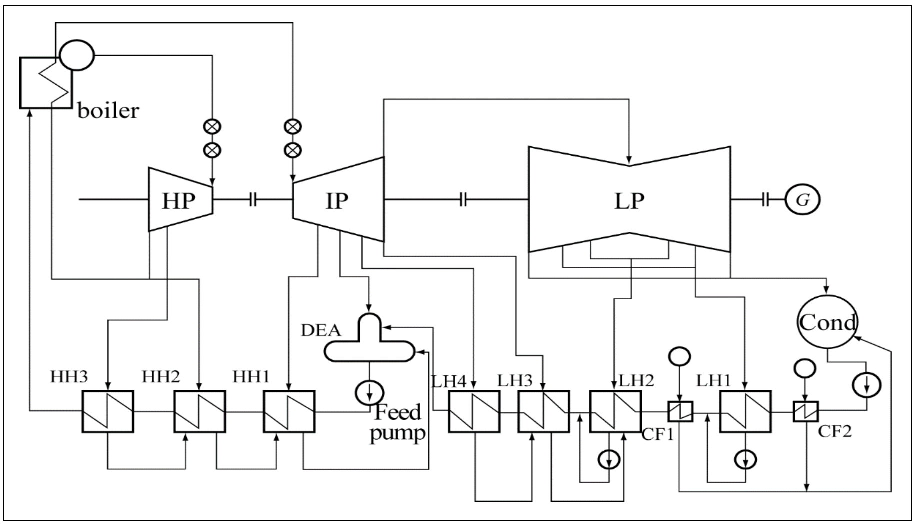

Fig 1: Main thermal system of 210MW reheating condensing turbine

210MW reheating condensing steam turbine is under consideration. This turbine has 3 cylinders, such asHigh Pressure Cylinder (HP), Intermidiate Presure Cylinder (IP) and Low Pressure Cylinder (LP). There are some heat exchangers, such as 3 High Pressure Heaters (HH1~HH3), 4 Low Pressure Heaters (LH1~LH4), deaerator and condenser.

The start-up data of the 210MW unit during the cold start-up process were provided from power plant shown as Fig.2.

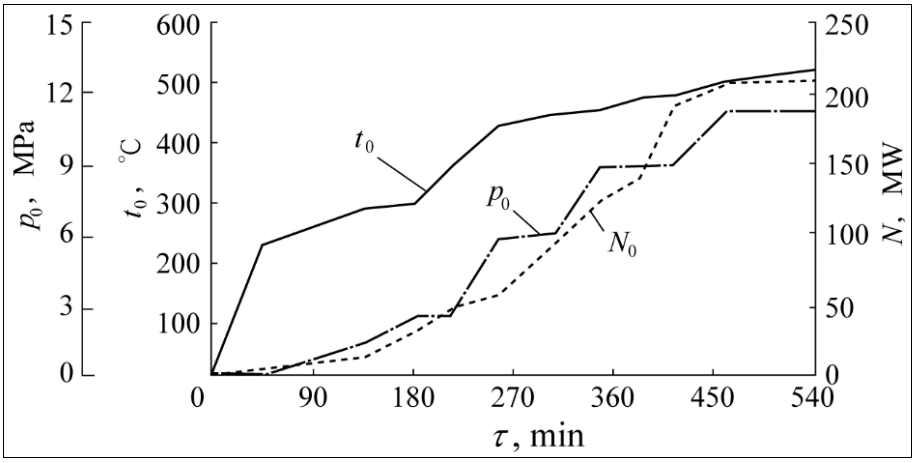

Fig 2: The cold start-up curve of the turbine

The temperature and pressur of the steam increased gradually with the initial ignition of the boiler at the second hour. When the main steam temperature reached 260~280℃ and the main steam pressure reached 1.8~2.0MPa, the turbine started rotating. The main steam temperature rise rate was 2.1℃/min until the revolution reached 500r/min. keeping the revolution for 10min, friction was checked. When the main steam temperature reached 350℃, the temperature rise rate increased (the intermediate speed heating). After the revolution reached 3000r/min, the unit began to grid connected. And the main steam temperature rise rate rose in the range of 1~1.5℃/min. Within 5 hours, the main steam temperature rose to the rated temperature 535℃ with various temperature rise rate. It is clear that increasing the temperature rise rate saves the start-up time and fuel consumption of this period causing the increase of power plant efficiency. But by the excessive rise rate of temperature or pressure, thermal stress and fatigue life loss could be increased in such parts as rotor. So the rise rates of temperature and pressure should be in the limited range, this is more serious for HP and IP rotor.

Moreover, this turbine was built in the term of 1983~1990. After the operation test, it had been used as a turbine in charge of main load of grid for 24 years till 2014. Since 2018, it has been serving as a unit in charge of peak load of grid.

During the long term operation, rotor of the turbine has sufferd from thermal fatigue loss by the start-up and shut down. So it is necessary to apply an optimized schedule for its start-up in order to ensure the efficient and safe operation, and exert its latent ability as a unit in charge of peak load

Fatigue suffering from the start-up and shut down process is called as low cycle fatigue (LCF), it is concerned to the maximum Von-mises stress [19].

The steam turbine start-up schedule can be composed to shorten the cold start-up time and decrease the maximum Von-mises stress s_{max}. Similar to the method proposed by Dong-Mei *et al.* [19], the start-up scheduling problem can be formulated as a multiobjective optimization problem with constraints by equation (1).

$$\min: f = f(\tau, \sigma_{\max}) \quad (1)$$

Subject to $\sum_{i=1}^{n} k_i \tau_i = \tau_{max}$

$\sigma_{\max} \leq [\sigma]$

The model of steam turbine off-design regime calculation

- It is necessary to calculate the temperature field of the rotor, in order to determine the thermal stress at the risk point. The temperatures at any points of the steam space can be determined by directly measuring with thermocouple or by calculating with the mathematical model.
- In general, it is almost impossible to mount the sensors on the points under consideration because of design limitation or the error of measuring [20].
- For the units which are impossible to measure, or absent of necessary sensors, unknown parameters can be calculated by known parameters.

The model of parameter calculation in steam turbine cascade: During the start-up process, steam turbine works on the off-design regime which differs from the design regime. So the steam parameters on each side of disks are changed with time. And in this process, to calculate the thermal stress of rotor, fore and rear parametrs of disks, i.e. outlet paramters of nozzles and blades in each stage are necessary. These parameters can be got by the off-design regime calculation.

According to the off-design regime calculation theory of steam turbine, there are two kinds of methods to determine the performance of turbine in off-design regime, such as simplified calculation and detailed one [22].

The simplified calculation is mainly used to analyse the performance of turbine unit in macroscopic viewpoint, by considering the steam turbine as an element of power plant. In this method, calculation is done by stage group, the parameters at the only fore and rear positions of stage groups are calculated, such as at the extraction room. Based on those parameters, the efficiency and output of unit are determined.

The kernel of this calculation is the relationship between flow and inlet pressure of stage group on the off-design regime, which was proposed by Stodola *et al.* [21], such as

$$\frac{G}{G_0} = \sqrt{\frac{p_{01}^2 - p_{z1}^2}{p_{00}^2 - p_{z0}^2}}\sqrt{\frac{T_{01}}{T_{00}}} \quad (2)$$

Inlet pressure of stage group can be calculated with flow by eq (2).

Compared with this, the detailed calculation is mainly used to analyse the steam turbine unit itself, by considering the stage as the element of calculation. The study subjects of this calculation are the variations of steam parameters at the nozzle and blade, and of temperature field in rotor and axial force led by it [22].

To calculate the thermal stress of rotor more correctly, the detailed calculation is more useful to determine the steam parameters at the fore and rear of each disk, i.e, at the outlets of nozzle and blade.

In many cases, the detailed calculation is progressed from the rear state to the fore state of the stage, inversely. This kind of detailed calculation is called as *Inversed calculation* [22]. Fig.3 shows the scheme of the *Inversed Calculation* method for the i^{th} stage.

As shown in Fig.3, the kernel of this calculation is the combination of continuity equation and iteration.

Flow variation causes the variation in efficiency, and makes the steam parameters change at extraction room. This affects the work of regenerative heaters, so that the extraction steam flow rate of those heaters get changed and the flow rate at the next stage group also change. So for the detailed calculation of off-design regime, the combination of cascade calculation and regenerative heating system calculation is needed.

I.e, the base parameter of detailed calculation of off-design regime, is flow rate of stage or stage group, and it can be determined by combining of cascade heat calculation and regenerative heating system calculation.

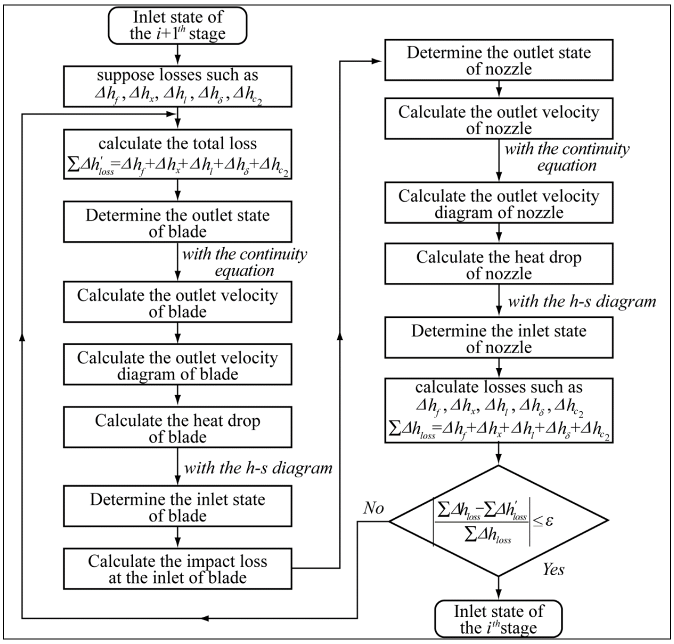

Fig 3: Scheme of the *Inversed Calculation* method for the i^{th} stage

The model of parameter calculation in regenerative heaters: There are two kinds of heat exchangers used for the regenerative heating system of steam turbine, such as mixed heat exchanger and surface heat exchanger [24]. Common model of the surface heat exchanger can be expressed as shown in Fig.4. *a*.

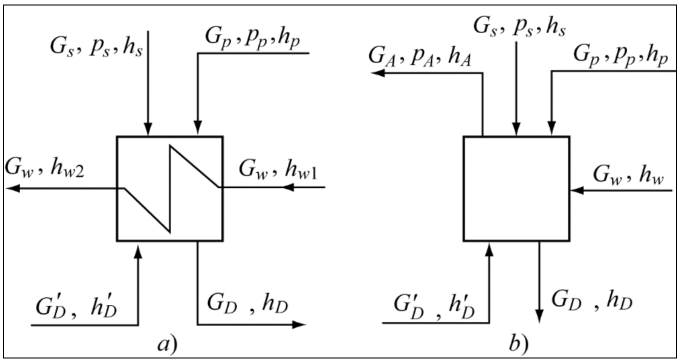

Fig 4: Common models of the heat exchanger

***a*) Surface heat exchanger, *b*) mixed heat excanger**

Energy balance and material balance equation for surface heat exchanger are

$$\begin{cases} G_s h_s + G_p h_p + G_w h_{w1} + G'_D h'_D = G_w h_{w2} + G_D h_D \\ G_D = G_s + G_p + G'_D \end{cases} \quad (3)$$

And the common model for mixed heat exchanger (deaerator) can be expressed as shown in Fig.4. *b*.

Energy balance and material balance equation for mixed heat exchanger are

$$\begin{cases} G_s h_s + G_p h_p + G_A h_A + G_w h_w = G_D h_D \\ G_D = G_s + G_p + G_A + G_w \end{cases} \quad (4)$$

By the cascade calculation, pressures and enthalpys of heating steams (extraction steams) supplied to each heat exchanger can be determined. And by the regenerative heating system calculation, flow rates, pressures and enthalpys of steams entering or exiting from component to component in the system are determined. So G_s (flow rate of extraction steam), h_{w2} (outlet enthalpy of feedwater or condending water) for surface heat exchanger and G_s for the mixed heat exchanger are determined by eq. (3), (4).

Flow rates and other paramters of water and steam passing through the components in turbine system can be determined by combining the relationships shown in Fig.3. and Fig. 4 to all the stages of turbine and heat exchangers in Fig.1. Water and steam properties are determined by IAPWS-IF97 [23].

The calculation model of steam paramters in individual stages of turbine, composed by combining of above two models can be expressed as shown in Fig.5.

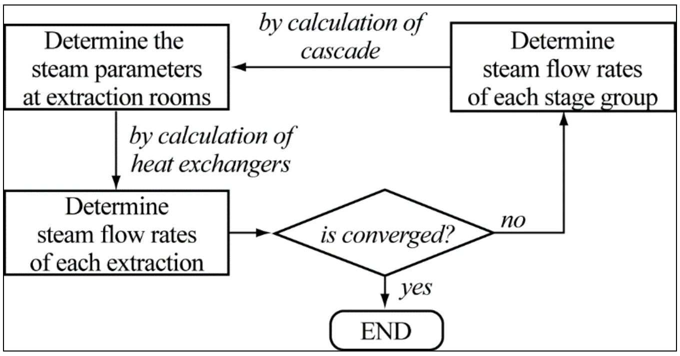

Fig 5: Steam turbine calculation model combined two models

Based on this model, a specific program was composed with VC, to calculate the off-design regime of 210MW steam turbine.

To verify this specific program, calculation results of this program was compared with the data [27] from the manufacturer, under the condition that turbine generates 50%, 60%, 70%, 75%, 80%, 90%, 100% and 105% of power to the design regime.

Table 1 and Table 2 shows the comparision results

Table 1: Calculation results compared to the provided data (Power)

Power, % / items	50	60	70	75	80	90	100	105
Provided data, MW	105	126	147	157.5	168	189	210	221.7
Calculation result, MW	107.7	127.6	146.9	156.7	165.3	184.9	209.2	222.2
Error, %	2.6	1.2	0.1	0.5	1.6	2.2	0.4	0.2

As shown in tables, the differences between calculation results and provided data is less than 5%.

Table 2: Calculation results compared to the provided data (Parameters of extraction steam)

Pow-er, %	Items	Ext.1			Ext.2			Ext.3			Ext.4			Ext.5			Ext.6			Ext.7			Ext.8		
		Press-ure, MPa	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**	**Press-ure, MPa**	**Flow rate, t/h**	**Enth-alpy, kJ/kg**
105	Calculated result	3.73	33.2	3150	2.4	33.3	3047	1.27	26.97	3370	0.698	12.5	3229	0.425	11.71	3109.9	0.256	15.09	2998.6	0.149	23.4	2902	0.047	29.7	2703
	Provided data	3.87	33.3	3148.9	2.507	33.5	3047.6	1.237	26.78	3380.6	0.685	12.55	3216.4	0.425	11.8	3096.2	0.253	15.14	2884.1	0.153	23.67	2884.1	0.0479	29.42	2687.4
	Error, %	3.62	0.30	0.04	4.27	0.60	0.02	2.67	0.70	0.31	1.9	0.40	0.39	0	0.77	0.44	1.19	0.33	3.97	2.61	1.15	0.62	1.88	0.94	0.58
100	Calculated result	3.6	30.6	3145	2.352	39.4	3049.7	1.197	24.82	3392.8	0.659	11.91	3248.5	0.402	10.67	3128.3	0.242	14.47	3015.1	0.141	21.75	2917.5	0.045	26.79	2714.9
	Provided data	3.62	30.13	3139.9	2.353	39.96	3039.9	1.164	24.62	3393.2	0.645	11.64	3217.3	0.4	10.85	3097.1	0.238	14.05	2884.8	0.144	22.02	2884.8	0.045	27.15	2687.3
	Error, %	0.55	1.54	0.16	0.04	1.42	0.32	2.84	0.81	0.01	2.17	2.27	0.97	0.5	1.69	1.01	1.68	2.90	4.52	2.08	1.24	1.13	0	1.34	1.03
90	Calculated result	3.39	24.8	3139.8	2.2	35.8	3057.2	1.062	20.61	3396	0.586	10.3	3251.8	0.358	9.5	3131.4	0.216	12.13	3018	0.126	19.8	2929	0.04	23.09	2717.4
	Provided data	3.229	25.32	3126.3	2.108	34.5	3028.5	1.045	21.17	3384.6	0.578	10.1	3220.2	0.359	9.3	3100	0.214	12.23	2980.5	0.13	19.13	2887.7	0.0407	23.65	2690.5
	Error, %	4.99	2.10	0.43	4.99	3.63	0.95	1.63	2.72	0.34	1.38	1.94	0.98	0.28	2.11	1.01	0.93	0.82	1.26	3.08	3.38	1.43	1.72	2.43	1
80	Calculated result	2.805	20.2	3109.8	1.852	29.6	3019.2	0.928	17.53	3351.2	0.511	8.95	3270.9	0.312	7.62	3094.7	0.188	10.35	2985.5	0.11	16	2890.4	0.035	20.07	2694.4
	Provided data	2.83	20.78	3111	1.856	29.15	3015.2	0.92	17.85	3387.1	0.511	8.59	3222.7	0.317	7.82	3103.3	0.189	10.47	2982.9	0.113	16.37	2890.1	0.036	20.14	2692.4
	Error, %	0.88	2.87	0.04	0.22	1.52	0.13	0.87	1.83	1.06	0	4.02	1.47	1.58	2.62	0.28	0.53	1.16	0.09	2.65	2.31	0.01	2.78	0.35	0.07
75	Calculated result	2.619	18.13	3099.3	1.733	26.09	3010	0.877	16.3	3377.5	0.4835	7.96	3234.7	0.296	7.26	3116	0.178	4.54	3004.7	0.104	15.55	2908.4	0.033	18.45	2707.8
	Provided data	2.645	18.77	3109.8	1.738	26.58	3012.5	0.862	16.35	3246.4	0.479	7.88	3223.9	0.298	7.14	3103.7	0.177	4.66	2984.1	0.108	15.11	2891.3	0.0339	18.43	2693.5
	Error, %	0.98	3.53	0.34	0.29	1.88	0.08	1.74	0.31	4.04	0.94	1.01	0.33	0.67	1.65	0.4	0.56	2.64	0.69	3.7	2.83	0.59	2.65	0.11	0.53
70	Calculated result	2.436	16.28	3091	1.615	23.95	3002.8	0.818	14.35	3379.2	0.4515	7.05	3236.3	0.276	6.77	3117.8	0.167	8.54	3006.4	0.097	13.91	2910.1	0.031	16.97	2709.3
	Provided data	2.465	16.89	3105.7	1.622	24.31	3010.9	0.805	14.91	3240.3	0.447	7.2	3225.1	0.278	6.49	3104.8	0.166	8.89	2985.3	0.101	13.89	2892.4	0.0317	16.75	2694.5
	Error, %	1.18	3.75	0.47	0.43	1.50	0.27	1.61	3.90	4.29	1.01	2.13	0.35	0.72	4.14	0.42	0.6	4.10	0.71	3.96	0.14	0.61	2.21	1.30	0.55
60	Calculated result	2.077	13.62	3072.4	1.384	19.81	2986.5	0.703	13	3381.6	0.3883	6.29	3238.4	0.238	4.8	3120	0.144	7.6	3008.6	0.084	11.51	2912.4	0.027	13.99	2711.2
	Provided data	2.111	13.49	3103.8	1.392	19.85	3009.5	0.691	12.93	3391.4	0.384	6.09	3227.1	0.24	4.92	3107.1	0.143	7.27	2987.5	0.0873	11.31	2894.9	0.0274	13.49	2696.6
	Error, %	1.61	0.95	1.01	0.57	0.20	0.76	1.74	0.54	0.29	1.12	3.18	0.35	0.83	2.50	0.41	0.7	4.34	0.71	3.78	1.74	0.61	1.46	3.57	0.54
50	Calculated result	1.728	10.73	3054.8	1.157	15.11	2971.1	0.589	10.48	3384.8	0.3258	4.61	3241.3	0.2	3.76	3122.7	0.121	5.71	3011.1	0.071	9.5	2914.9	0.023	10.11	2712.9
	Provided data	1.765	10.37	3105.7	1.166	15.67	3011.5	0.58	10.23	3393.8	0.323	4.73	3229.7	0.202	3.87	3109.7	0.121	5.88	2990.1	0.0736	9.13	2897.4	0.0232	10.32	2698.9
	Error, %	2.1	3.36	1.64	0.77	3.71	1.34	1.55	2.39	0.27	0.87	2.60	0.36	0.99	2.93	0.42	0	2.98	0.7	3.53	3.89	0.6	0.86	2.08	0.52

Finite element calculation of rotor

In this paper, SIMULATION module of SOLIDWORKS2018 was chosen to calculate the rotor of reasonable simplification.

The finite element analsys and model of rotor

In the calculation of the transient temperature field of turbine rotor, the rotor is considered a homogeneous, isotropic and no heat source object. Due to the structural features of turbine rotor, 2D analysis can be used for the analysis of the rotor. Some parts of the rotor were simplified, such as labyrinth seal, in order to save the cost and computational resources.

Due to the effects of the centrifugal force on blades to the stress of rotor, the blades can be transformed to the centrifugal force. I.e, blades and shrouds are shifted to annulars, combined with blade wheel, which have the same widths with disks. And the radial heights of the annulars can be determined in order to have the same centrifugal force with the sum of blades and shrouds [25].

Centrifugal force of the leaned blade

$$C_d = mR_p\omega^2 \tag{7}$$

The centrifugal force of this annular is

$$dc = \omega^2 Rdm = \rho\omega^2 RyRd\varphi dR = \rho\omega^2 R^2 yd\varphi dR \tag{8}$$

Integrating the above equation to the whole annular

$$c = \iint_{\Omega} \rho\omega^2 R^2 yd\varphi dR = \rho\omega^2 y\int_0^{2\pi} d\varphi \int_{R_1}^{R_2} R^2 dR = \frac{2}{3}\pi\rho\omega^2 y\left(R_2^3 - R_1^3\right) \tag{9}$$

Due to the equivalent in centrifugal forces, $c=C_d$, so

$$R_2 = \sqrt[3]{\frac{3mR_p}{2\pi\rho y} + R_1^3} \tag{10}$$

After the equivalent converting, the blade tip radiuses of all stages in HPC (high pressure cylinder) will change as shown in Table 3.

Table 3: The original radiuses and equivalent radiuses in high pressure cylinder (HPC)

Stage No.	1	2	3	4	5	6	7	8	9	10	11	12
Original radius, m	0.4670	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955	0.3955
Equivalent radius, m	0.494	0.44	0.442	0.443	0.447	0.449	0.451	0.454	0.456	0.455	0.456	0.458

The triangluar element which has 6 nodes is used as 2D element, in SIMULATION of SOLIDWORKS. The number of nodes in the finite element model was 32649, and the number of elements was 14374, as shown in Fig.6.

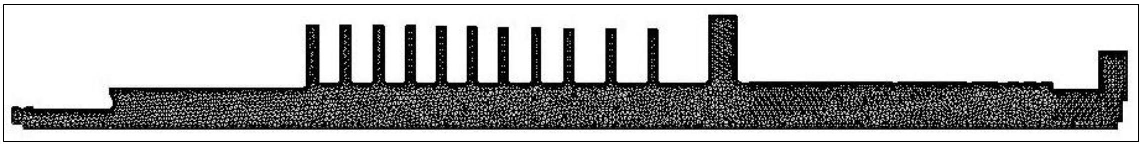

Fig 6: The finite element model of the rotor

The material of the rotor is 30Cr1Mo1V steel, and its properties were shown in Table 4 and 5. [19].

Table 4: The mechanical properties of 30CR1MO1V steel to the temperature

Temp, °C	Yield strength s_s, MPa	Strength limit s_b, MPa	Elongation *d*, %	Reduction of area *y*, %
20	629	779	20	60
540	465	520	29.6	88.5

Table 5: The material properties of 30Cr1Mo1V steel to the temperature

Physical Features	Temp, °C						
	20	100	200	300	400	500	600
Young modulus *E*, GPa	214	212	205	199	190	178	178
Poission ratio *m*	0.288	0.292	0.287	0.299	0.294	0.305	0.305
Thermal conductivity l_s, W/(mK)	48.5	47.1	44.8	42.8	40.3	37.5	35.3
Specific heat *c*, J/(kgK)	554	574	599	624	666	720	824
Linear expansion coefficient a_l, 10^{-6}1/K	0	11.99	12.81	13.25	13.66	13.92	14.15

1. Boundary condtions

Thermal boundary conditions

1. Because of cooling by lubrication, the temperature on the journal surface is relatively stable. So the first order boundary condition is set on the journal surface, i.e. constant temperature of 70°C.
2. The heat transfer coefficients between left and right end of rotor and air are small, so the insulation condition is applied on them. And the surface at the center hole, there is no any heat exchange, so the insulation condition is applied on it, too.
3. On the outer surface of the rotor, the third order boundary condition is applied, determined by steam temperature and heat transfer coefficient in given region.
4. Steam parameters differ through the packing groups, and ends of packing boxes are connected to the other elements (such as regenerative heaters, packing steam cooler as shown in Fig.7) of system, so the third order boundary condtion is applied on the packing. These

parameters are determined by the program mentioned above.

Based on the parameters, the third order boundary condition is applied on each packing group, by defining heat transfer coefficient and environment temperature.

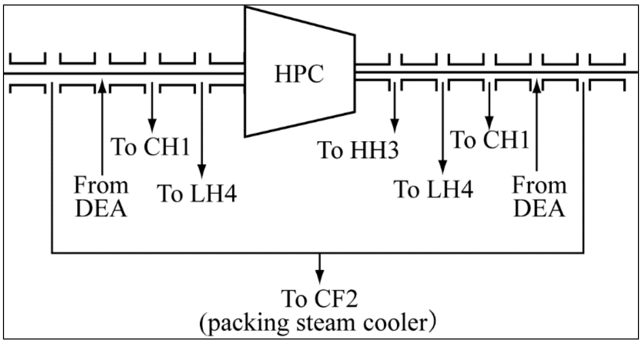

Fig 7: Packing steam system of HPC

2. Structural boundary condition: There is a thrust medal at the left end of rotor, so U_x, the displacement in x direction on the left end is 0.

Heat transfer coefficient calculation: Steam parameters and heat transfer coefficients differ from point to point on the rotor surface according to the temperature. So the variations are the functions of space-time [19]. So, at first, steam paramters are calculated by the program mentioned above, and then *l*, thermal conductivity, *m*, dynamic viscosity coefficient and *v*, the specific volume of steam are determined by IAPWS-IF97, to calculate the heat transfer coefficient. Through a lot of experiments and theoretical analysis the heat transfer coefficient on each surface of rotor have been studied. In this study, Haqi-Nangong formula [19] was employed.

1. Heat transfer coefficient on both sides of the disk

$$\alpha = \frac{Nu \cdot \lambda}{R_b}, \mathrm{W/\left(m^2 \cdot K\right)} \tag{11}$$

When $Re \le 2.4 \times 10^5$, $Nu = 0.675Re^{0.5}$ and when $Re > 2.4 \times 10^5$, $Nu=0.0217Re^{0.8}$

Where $Re=u_b R_b/\nu$, R_b is the outer radius of the disk, in m, *l* is the thermal conductivity of steam, *n* is the kinematic viscosity of steam, u_b is the circumferntial velocity at R_b, in m/s.

2. Heat transfer coefficient on the surface of axis

$$\alpha = \frac{Nu \cdot \lambda}{R_a}, \mathrm{W/\left(m^2 \cdot K\right)} \tag{12}$$

Where $Re=u_a R_a/n$, $Nu=0.1Re^{0.68}$, R_a is the radius of the axis at the position under consideration in m, $u_a$is the circumferential velocity on the shaft journal in m/s.

3. Heat transfer coefficient at the labyrinth seal

$$\alpha = \frac{Nu \cdot \lambda}{2\delta}, \mathrm{W/\left(m^2 \cdot K\right)} \tag{13}$$

Where, *d* is the clearance of labyrinth, in m, W_z is the average velocity of steam through the steam sealing clearance, in m/s, *H* is the height of rotor surface to sealing ring, in m, S is the width of sealing chamber, in m.

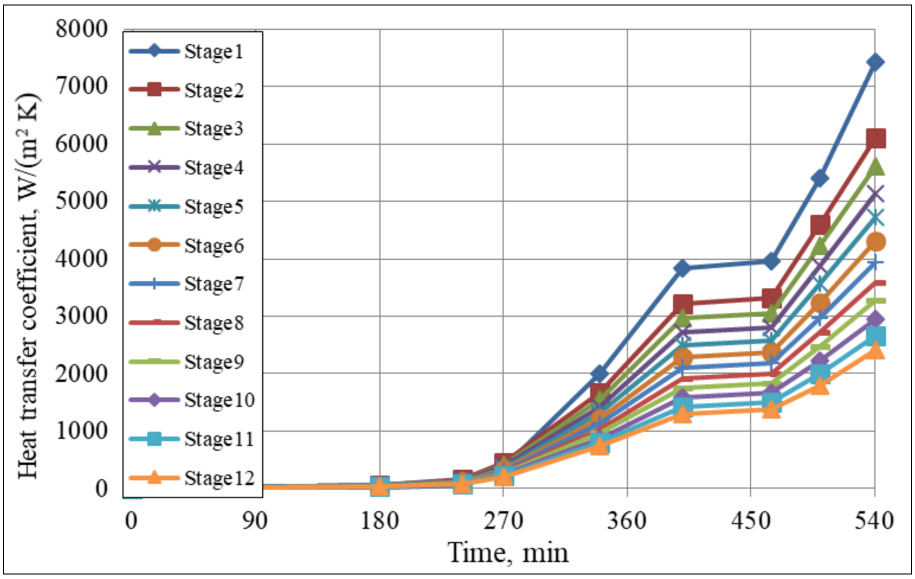

Fig 8: The variation of heat transfer coefficients at the right sides of each disk to time

According to the main paramters of the start-up schedule in each interval, the heat transfer coefficients of all parts were calculated repectively, and the steam temperature and pressure of steam were applied on the outer surface as boundary conditions. The heat transfer coefficients at the right sides of each disk (conformed to the outlets of nozzles) are shown in Fig.8.

Stress field of rotor calculation

The rotor being considered has a center bore of which diameter is 100mm.

To calculate the stresss field, *Thermal* and *Nonlinear* modules of SIMULATION in SOLIDWORKS were employed. According to the FEA results of rotor in a start-up schedule, the dangerous regions were at the transition from the fore side of govern stage to the front end labyrinth, the surface of center bore under the govern stage disk and the rear of govern stage disk. So the monitoring points were choosen at points A, B and C as shown in Fig.9.

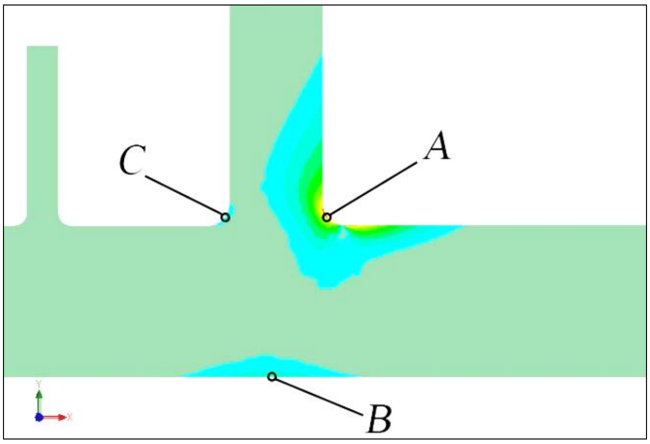

Fig 9: Monitoring points of rotor

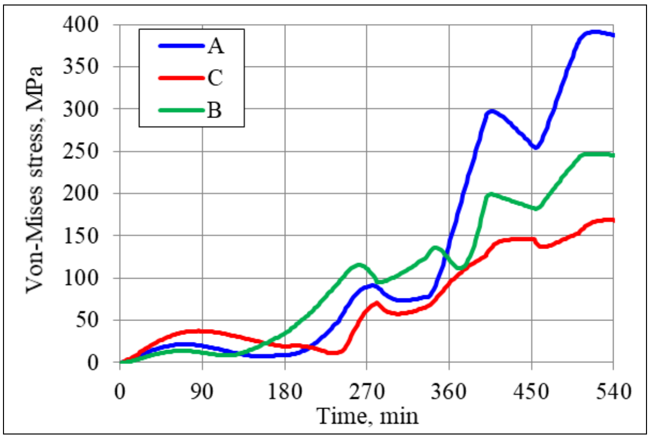

Fig 10: The variation of stresses at the monitoring point's variation to time

The monitoring result for those points, were shown in Fig.9. The maximum Von Mises stress over the whole cold start-up process is at the point A, the value is 397MPa.

Combining the result with the start-up curve shown in Fig.2, the temperature gradient starts rising in the interval which the temperature rise rate increases, causing the increase in stress.

Optimization of cold start-up schedule

Sample data of cold start-up schedule: Thermal stress of rotor is propotional to the temperature gradient, and the steam temperature variation affects the stress of rotor. So, to reduce the cold start-up time, the temperature rise rate must be improved so as to satisfy the condition that the stress rotor must lower than the allowable stress. And the duration of the temperature rise rate affects the whole start-up time, moreover the fuel consumption and the whole power plant efficiency.

So, in this study, ten start-up schedules were composed by varying the temperature rise rates and their durations in each interval of cold start-up process. According to those schedules, the maximum Von-Mises stresses were determined by FEA. To simplify the operation process, variations were allowed up to eleven. For each schedule, k_i, t_i (i=1..11) are the temperature rise rate in °C/min, and its duration, in min.

The maximum Von Mises stresses of rotr to the cold start-up schedules are shown in Fig.6. Here, schedule 1 is the original start-up schedule.

Table 6: Main parameter and the maximum von-mises stress of cold start-up schedules

Sche-dule	k_1, °C/min	t_1, min	k_2, °C/min	t_2, min	k_3, °C/min	t_3, min	k_4, °C/min	t_4, min	k_5, °C/min	t_5, min	k_6, °C/min	t_6, min	k_7, °C/min	t_7, min	k_8, °C/min	t_8, min	k_9, °C/min	t_9, min	k_{10}, °C/min	t_{10}, min	k_{11}, °C/min	t_{11}, min	t, min	s_m, MPa
1	0.75	40	0.7	100	0	40	1.63	80	0.4	90	1	20	0	60	3.5	20	0.1	90	0	0	0	0	540	397
2	0.583	120	0.833	72	1.0	30	0.857	70	0.667	60	1.077	65	0.571	35	0.125	40	0	0	0	0	0	0	492	392.9
3	0.425	120	1.750	54	0.0	30	1.677	65	0.0	65	1.415	53	0.0	47	0.625	40	0	0	0	0	0	0	474	397
4	0.300	120	1.567	54	0.0	30	0.646	60	1.585	60	1.415	55	0.0	40	0.625	37	0	0	0	0	0	0	456	423.6
5	0.383	100	1.567	50	0.0	30	0.646	60	1.585	65	1.415	50	0.0	50	0.625	42	0	0	0	0	0	0	447	436.6
6	0.217	130	1.567	45	0.0	25	0.646	70	1.585	60	1.415	60	0.0	42	0.625	50	0	0	0	0	0	0	482	426.6
7	0.217	150	1.567	50	0.0	41	0.778	53	1.585	65	1.415	53	0.0	47	0.625	40	0	0	0	0	0	0	499	387.2
8	0.900	80	0.800	55	0.146	41	1.074	68	0.692	70	0.811	41	0.681	58	0.075	30	0.367	100	0	0	0	0	543	385.8
9	0.500	115	0.117	65	1.756	60	1.407	60	0.246	65	0.189	65	0.681	50	0.075	42	0.367	70	0.100	60	0.050	63	715	376.5
10	0.292	110	0.850	100	1.293	50	0.889	54	0.677	65	0.604	53	0.340	47	0.150	50	0.217	60	0.150	60	0.100	50	699	365.6
11	0.292	100	0.850	75	0.073	20	0.444	60	0.354	70	0.849	56	1.468	50	0.175	55	0.267	55	0.200	70	0.500	100	711	360.8

Regresion of sample data

Regression models express the relationship between responses (outlet) and one or more inputs (inlet).

SVM is a machine learning method proposed by Valdimir Vapnik *et al.* [26] in 1992.

SVM not only has a goon nonlinear processing ability, but also avoids the over learning of the neural network. Therefore, it is widely used for pattern recognition, regression analysis and life estimation. MATLAB 2018 provides SVM (e-SVM) tool in the *Static and Machine Learning Toolbox*. Training samples includes inputs and responses. The main goal of SVM is to define f which y, determined by y=f(x), is less than e.

Many kernel functions are used such as Gaussian kernel, Polynomial kernel and sigmoid kernel. Gaussian kernel function is widely used for practical problems seen as Eq.14.

$$G\left(x_j, x_k\right) = \exp\left(-\left\| x_j - x_k \right\|^2\right) \qquad (14)$$

As shown in Table 6, in general, the shorter the start-up time is, the higher the maximum Von-Mises stress. But comparing schedule 1 with 3 in Table 6, though the start-up time is short, there is no variation of maximum Von-Mises stress, according to the selection of start-up parameters. The start-up times of schedule 1 and 3 are 540min and 474min, respectively. The start-up time of schedule 1 is longer than of schedule 3, but the maximum Von-Mises stresses are 397MPa.

The proposed eleven schedules, in Table 6, were used as training samples, to establish the SVM regression model. The SVM model was established with the penalty parameter *C* was 5.0121, the insensitive loss function *e* was 0.06079 and the width parameter of the Gaussian kernel function *g* was 0.00612. Mean square error and squared correlation coefficient of the model for the training set are 0.0012, 0.9942, respectively. The actual stress value (results of FEA) and the prediction value (results of regression) of model are shown in Fig.11.

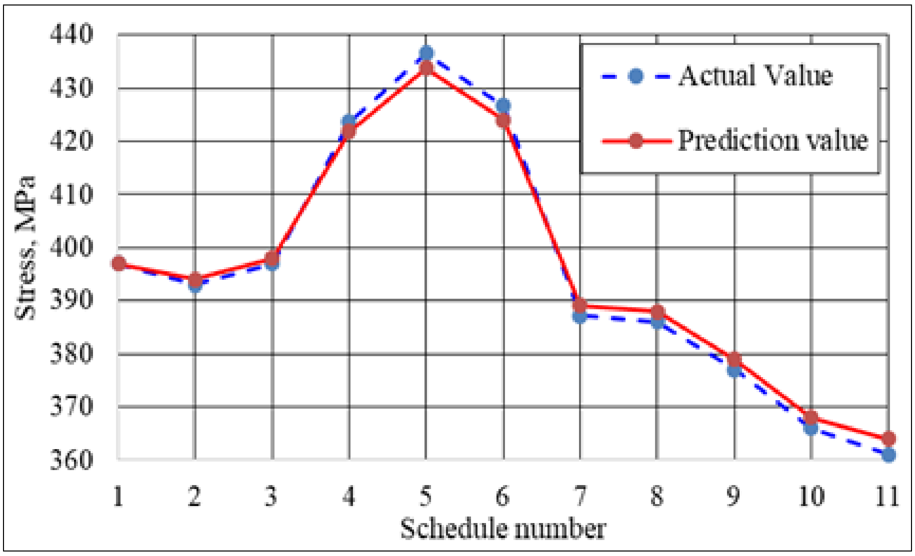

Fig 11: The actual stress value and the prediction value of SVM model

Table 7 shows the prediction values, actual values and their relative errors.

Table 7: Prediction values, actual values and their relative errors

Schedule No.	1	2	3	4	5	6	7	8	9	10	11
Actual value, MPa	397	393	397	423.6	436.6	426.6	387.2	386	377	366	361
Prediction value, MPa	397	394	398	421.81	433.78	423.96	389.13	388	379	368	364
Relative error, %	0.09	0.31	0.14	0.423	0.645	0.619	0.498	0.53	0.75	0.69	0.78

As seen in the table, the prediction error of the model is only 0.78%, and this verifies the accuracy of the model.

4.3 Optimization of start-up schedule with GA

As eq. (1) mentioned above, to increase the lifetime of rotor which has been used for a long period, is the purpose of this study.

Genetic Algorithm (GA) is an optimization algorithm which simulates the biological evolution, and widely used for the solution of discontinuous, underivable, strong non-linearity problems.

MATLAB provides GA tool in *Optimization*.

The decision variables proposed in this study are k_i, the temperature rise rate in each step of the start-up process, t_i, the duration of the given temperature rise rate (i=1, …, 11).

As mentioned above, the objective function f should be determined in order to demonstrate the latent ability of the unit by ensuring not only the efficiency but also the safety. So, expressing the eq. (1) again,

$$\min: f = f(\tau, \sigma_{max}) = \frac{\tau}{\tau_{base}} + \frac{\sigma_{max}}{[\sigma]} \qquad (15)$$

The first term of this equation expresses the normalized start-up time, and the second term is for the maximum stress.

The base time t_{base} is chosen as 540min, the allowable stress [*s*] is chosen as 450MPa by considering the yield stress and strength limit from Table 4 and the maximum steam temperature t_{max} is chosen as 535℃ from the requirement of system.

For the optimization, in GA, floating point number encoding was used to encode the decision variables, the maximum generation was 200, the population size was 150, uniform selection, arithmetic crossover (fraction 0.4, coefficient 0.4) and Gaussian Mutation (fraction 0.01) were employed.

The best fitness corresponding to the optimized solution was 1.63675, it was appeared at the near of the 195th generation as shown in Fig.12. The values of decision variables were shown in Table 8.

Table 8: The values of decision variables

Step No. Items	1	2	3	4	5	6	7	8	9	10	11
Temperature rise rate, ℃/min	0.724	1.081	1.838	1.4	1.534	0.195	0.501	0.531	0.758	0.133	0.257
Duration, min	49.84	35.999	34.284	41.374	34.592	43.298	35.955	30.149	37.496	30.154	43.01

For the optimal schedule, the maximum Von-Mises stress was 391. 39MPa, the start-up time was 416.15min.

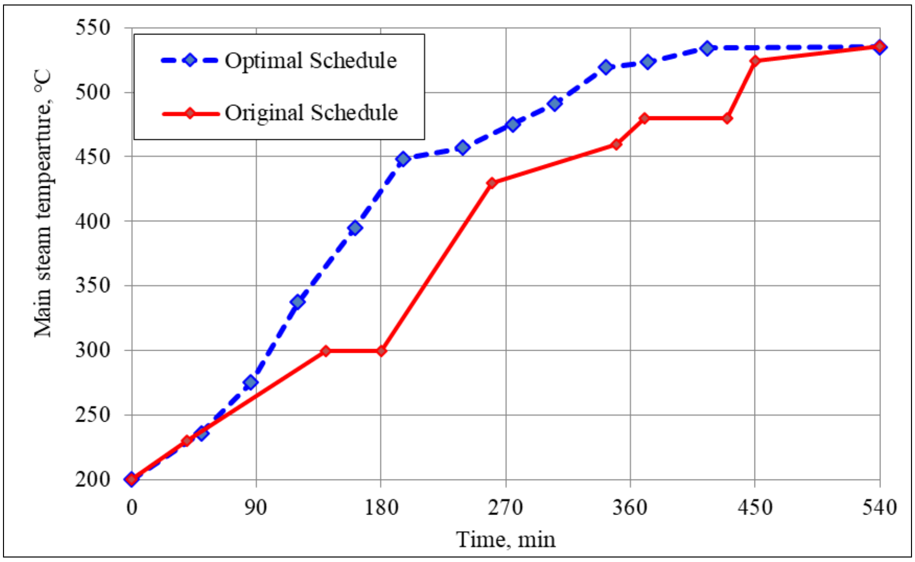

Fig 12: The variations of main steam temperatures under the optimal schedule and the original schedule

The steam temperature reached the maximum value in 540 min under the original start-up schedule, but 416min under the optimal schedule, as shown in Fig.13.
Next, the increament of fatigue lifetime at the optimal schedule, is considered.
There is a relationship between cyclic stress and strain as follow [14].

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'} \tag{16}$$

And there is a relationship between the total strain amplitude and the fatigue life time by the classical Coffin-Manson equation [15].

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{17}$$

The factors to determine the fatigue lifetime of 30CrMoV, the rotor material were shown in Table9.
The increasing rate of fatigue lifetime according to the reduction of the maximum Von-Mises stress from 397MPa to 391.39MPa can be determined by eq. (17).

$$\frac{N'_f}{N_{f0}} = 1.194 \tag{18}$$

As this, the lifetime increases as much as 19.4%, under the optimal schedule.

Table 9: The factors to determine the fatigue lifetime of 30CrMoV

Temperature, °C	σ'_f **, MPa**	ε'_f	***b***	***c***
25	1008	0.6284	-0.0804	-0.825
510	680	0.3884	-0.0856	-0.716
538	588	0.6264	-0.0697	-0.755

Verification of the optimal start-up schedule
Based on the optimization results, the thermal stress of the rotor in the optimal start-up schedule was analysed by *Thermal* and *Nonlinear* modules of SIMULATION in SOLIDWORKS. Von-Mises stresses of three monitoring points under the cold start-up schedule were shown in Fig.14.

- The maximum Von-Mises stress was 394MPa, appeared at 330min.
- The relative error between prediction value(391.39MPa) and FEA result(394MPa) is only 0.86%, so the optimal schedule was verified.
- Under the optimal schedule, the time which the main steam temperature reached the rated value, was shorten by 124min.

Conclusions: With the gradually increase of the power grid capacity, and the difference between peak and valley values of the grid, high power steam turbines are frequently participated in peaking operation. To meet such requirement, the units charging of peaking load have to start, stop frequently. Therefore, the optimal cold start-up schedule of a 210MW subcritical steam turbine was studied in this study, in order to ensure not noly the efficiency and safety of the unit, but also the rapidity of start-up.

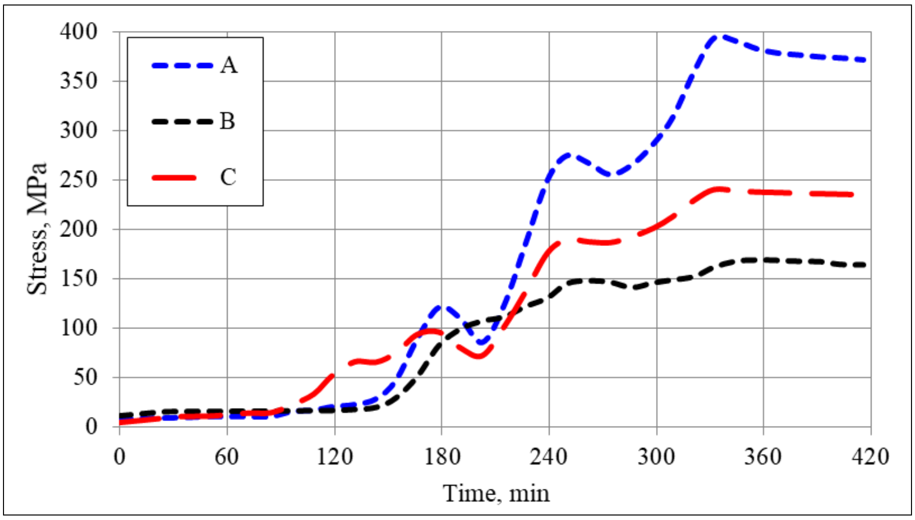

Fig 13: Von-Mises stresses of the three monitoring points under the optimal start-up schedule

Research works had been done as following

1. An optimization object function was proposed with k_i, t_i (i=1.11), respectively representing the temperature rise rate and its duration on each step to ensure the rapidity and increase the lifetime by reducing the start-up time and thermal stress.
2. A model to calculate the off-design regime was propsed to determine the steam parameters at each parts of cascade, such as outlet of nozzle and blade. This model was built by combining the off-design regime calculation theory and thermal system design theory.
3. Eleven cold start-up schedules were composed, and FEM was employed to calculate the stress field of rotor under the different schedules.
4. With the FEM results of eleven start-up schedules as sample data, SVM was employed to build the regression model of maximum Von-Mises stress under the differente start-up schedules.
5. Based on the regression model of SVM, GA was applied to find the optimal start-up paramters and start-up schedule of the unit.

The proposed optimization control scheme is based on the off-design calculation model, SVM model and GA. The optimal solution appears at the 195th, the fitness function value is 1.63675, which is minimum in the process of evolution. Results show that the present optimal control scheme can shorten the start-up time by about 23%(124min) with reducing the Von-Mises stress as much as 1.4%, increasing the fatigue lifetime 1.2 times as much as the original schedule. It will save energy significalntly and improve the lifetime of the turbine.

References

1. Ding YJ. Study on the optimization of *steam turbine* start-up scheduling. Zhejiang University; 2013.
2. Belkhri F, Kraus Cabo D, Felgner F, Frey G. Optimal startup control of a *steam* power plant using the JModelica platform. IFAC-PapersOnLine. 2015;48(1):204-209.
3. Tica A, Gueguen H, Dumur D, Faille D, Davelaar F. Hierarchical model predictive control approach for start-up optimization of a combined cycle power plant. Applied Energy. 2013;98:256-265.
4. Hubel M, Meinke S, Andren MT, Wedding C, Nocke J, Gierow C, *et al.* Modelling and simulation of a coal-fired power plant for start-up optimization. Applied Energy. 2017;10:1-13.
5. Shirakawa M, Kawai K, Arakawa M, Nakayama H. Intelligent start-up schedule optimization system for a thermal power plant. IFAC Symposium on Power Plants and Power Systems Control. 2006;189-194.
6. Ponce CV, Saez D, Bordons C, Nunez A. Dynamic simulator and model predictive control of an integrated solar combined cycle plant. Energy. 2016;109:974-986.
7. Celis C, Pinto GRS, Teixeira T, Xavier E. A *steam turbine* dynamic model for full-scope power plant simulators. Applied Thermal Engineering. 2017;1-9.
8. Lu G, Sun C, Lv W, Wang C, Xu B. Application on modeling and simulation of turbine generator in DMS2016 marine engineering simulator. International Conference on Measuring Technology and Mechatronics Automation. 2017;272-273.
9. Bausa J, Tsatsaronis G. Dynamic optimization of startup and load-increasing processes in power plants - Part I. Journal of Engineering for Gas Turbines and Power, Transactions of the ASME. 2001;123:246-250.
10. Martin J, Medina-Flores J, Picon-Nunez M. Modelling the power production of single and multiple extraction *steam turbines*. Chemical Engineering Science. 2010;65:2811-2820.
11. Lv Y, Wang P. Experimental study about optimal operating conditions of the *steam turbine*. North China Electric Power University; 2010. p. 1-4.
12. Zucca S, Botto D, Gola MM. Faster on-line calculation of thermal stresses by time integration. International Journal of Pressure Vessels and Piping. 2004;81:393-399.
13. Li Y, Wang J. Low-cycle fatigue behavior of 30Cr2MoV steel at elevated temperatures. International Journal of Fatigue. 1992;3:169-172.
14. Lv F, Wang K, Huang S. Low-cycle fatigue life estimation of *steam turbine* rotor steels at high temperature. The Journal of Strain Analysis for Engineering Design. 2013;49:19-26.
15. Sun YJ, Liu XQ, Hu LS, Tang XY. Online life estimation for *steam turbine* rotor. Journal of Loss Prevention in the Process Industries. 2013;26:272-279.
16. Banaszkiewicz M. On-line monitoring and control of thermal stresses in *steam turbine* rotors. Energy.

2016;94:763-776.
17. Lyubimov AA, Troitskii AI, Gladshtein VI. Analyzing the state of *steam turbine* metal based on safety and reliability factors. Thermal Engineering. 2013;60(1):32-37.
18. Ji DM, Sun JQ, Dui Y, Ren JX. The optimization of the start-up scheduling for a 320 MW *steam turbine*. Energy. 2017;17.
19. Chaibakhsh A, Ghaffari A. *Steam turbine* model. Simulation Modelling Practice and Theory. 2008;16:1145-1162.
20. Stodola A. *Steam and Gas Turbine*. Vol. 1. New York: Peter Smith; 1945.
21. Li WT, Huang BH. Thermal calculation of *steam turbine* on off-design regime. Chinese Electric Power Press; 2001.
22. The International Association for the Properties of Water and Steam. Release on the IAPWS International Formulation 1997 for the Thermodynamic Properties of Water and Steam. Erlangen, Germany; 1997.
23. O'Kelly P. Computer Simulation of Thermal Plant Operations. New York: McGraw-Hill; 2013.
24. Hao Z. Stress and strain thermal-structure interaction numerical analysis of 600 MW turbine rotor. Huabei Electrical University; 2013.
25. Vapnik V. The Nature of Statistical Learning Theory. New York: Springer; 1995.
26. Haerbin Steam Turbine Production Company. N210-130/535/535 *Steam Turbine* Thermodynamic Parameters. Haerbin Steam Turbine Production Company; 1987.