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Effect of the lean angle of blade leading edge on the pressure pulsation of a francis-130 turbine

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Abstract
The pressure distribution and velocity distribution in the runner, the flow regime and pressure pulsations inside the turbine is related to the lean angle of the runner blade leading edge.
In this paper, the effect of the lean angle of the runner blade leading edge on the pressure pulsation in the flow passage of Francis-130 turbine operating in off-design condition is investigated.
This paper presents the variation of the characteristics of the turbine, the pressure distribution and the pressure pulsation in the flow channel of runner, spiral casing, guide vanes and draft tube with the change of lean angle of the leading edge based on numerical simulation, and a suitable range of the lean angle of the blade leading edge is suggested.
The study concluded that the lean angle of Francis-130 turbine is 20° which is more suitable for improving the hydraulic turbine operating stability.

Keywords: Francis turbine, pressure pulsation, pressure distribution, lean angle of leading edge, off design condition

1. Introduction
In a hydraulic turbine, the runner is the main part of energy conversion, so its design is significant for the performance. The blades are important parts of the runner and reasonable design of the blade is essential for improving performance of a hydraulic turbine.
The shape of the runner blade is generally determined by the profiles of the leading edge and streamline. The leading edge has a straight or curved shape. In a Francis turbine with low specific speed, it has a straight shape, whereas it has a curved shape in a higher specific speed. The leading edge of the runner blade is leaned with respect to the axial direction, which effects the lean angle distribution of the blade and the pressure and velocity distribution in the runner flow channel.
The pressure gradient for any direction in the runner is related to the geometrical parameters and the velocity distribution as follows [1].

$$\frac{d\underline{h}}{dL} = 2\Bigg\{\left[\left(\frac{1}{R} - \frac{\cos\delta\cos^3\beta}{r}\right)\frac{\underline{c}_m^2}{\sin^3\beta} - \underline{c}_m\frac{\partial \underline{c}_m}{\partial m}\frac{1}{\tan\beta} + 2\underline{\omega}\cos\delta\,\underline{c}_m\right]\tan\theta + \left[\left(\frac{\sin(\delta-\psi)}{r\tan^2\beta} + \frac{\cos\psi}{\rho}\right)\underline{c}_m^2\right] + \left[2\underline{\omega}\frac{\sin(\delta-\psi)}{\tan\beta} - \frac{\partial\underline{c}_m}{\partial m}\sin\psi\right]\underline{c}_m + \underline{\omega}^2 r\sin(\delta-\psi)\Bigg\} \qquad (1)$$

In Eq. (1), h is the static pressure head, L is the direction of pressure gradient, m is the direction of streamline in the meridian section, R is the radius of curvature of streamline in the flow surface, r is the radial coordinate of the interesting point, ρ is the radius of curvature of streamline in the meridian section, δ is the angle between the direction normal to the stream surface and axial direction, β is the angle between the flow direction in the steam surface and the circumferential direction, ψ is the angle between gradient direction of the pressure and axial direction, θ is the lean angle of the blade, c_m is the velocity in the meridian section, ω is the angular velocity of the runner, and $\underline{h} = h/H$, $\underline{c}_m = c_m/\sqrt{2gH}$, $\underline{\omega} = \omega/\sqrt{2gH}$.

As shown in Eq. (1), the pressure gradient in any direction of the flow channel depends on the blade shape.

Hence, the pressure gradient in the direction normal to the streamline is changed with the change of blade lean angle, and the pressure pulsation property in the flow channel is also changed.

So far, many studies have been carried out to investigate the pressure pulsation characteristics in the flow channel. The studies can be classified into a method of measuring pressure pulsations in a flow channel of a prototype turbine using measurement tools and a method of predicting pressure pulsations at the points of interest in a flow channel based on numerical simulations.

The measurements of a prototype turbine are the performance assessments of the turbine designed and manufactured already, and the predictions of pressure pulsations by the numerical simulation are used as a means of improving the turbine performance at the design stage.

The study [2] presented the recent trends and ideas for flexible operation of Francis turbines using Full-Size Frequency Converter (FSFC) or Doubly-Fed Induction Machine (DFIM) technology for variable-speed operation. This technology allows for the speed of the runner to be adjusted in order to maximize the efficiency and/or reduce dynamic loads of the turbine according to the available head and power generation demands. Continuous speed variation of up to ± 10% of the design rotational speed can be achieved with the DFIM technology, while for FSFC there is no such limit by the technology itself. By setting the turbine's speed according to head and power conditions, the pressure pulsation is decreased and the operating stability of the hydraulic turbine is improved, it is because reducing the vortex in the flow passage between the blades.

In [3], FSI analysis of hydro-turbine with a hydro-power plant and concrete structures is carried out to simulate the hydraulic and mechanical interactions between them. Here, DES simulation method was applied to simulate the transient in the entire flow channel and thereby the pressure pulsation characteristics are predicted.

In [4], numerical simulations were carried out for three cases using the MRF model, the moving mesh model and considering the compressibility, and the results were compared. The pressure pulsation and pulsation spectra obtained by simulations and experiments were compared, the results showed that the simulations considering the compressibility and using the moving mesh model predicted the pressure pulsation more significantly than the other. It was also shown that the rotational motion of the vortex rope is the source of pressure pulsation in the entire flow channel.

In [5], a method for measuring characteristic frequency is proposed by experiments on a reduced scale model of a Francis turbine operating at off-design condition, and it is verified. The assessment of hydro power plants operating at off-design condition can be done using one-dimensional way, but a proper modeling method of the cavitation flow in the draft tube is necessary. The test rig is excited by injecting a periodical discharge with a rotating valve whose frequency linearly increases from 0 to 7 Hz. The pressure pulsation was measured by sensors installed in the flow passage and analyzed.

The flow and pressure pulsations in a Francis turbine draft tube operating in a wide operating range using a vortex generator and a computer measuring device were analyzed [6, 10, 11].

In [7], the pressure pulsations in a high head Francis turbine operating at variable speed are analyzed and suitable method of speed control is investigated. The study showed that increasing the speed at off-design operating condition leads to a further increase in the pressure pulsation amplitude.

In [8], the pressure pulsation measurements were carried out using pressure sensor in a vertical and horizontal Francis turbine, and their variation characteristics were analyzed. Further analysis showed that the amplitude of a frequency related to the draft tube vortex rope is within the limit of 3.5% to 5.3% of the head. In the vertical axis turbine, amplitudes of asynchronous pressure pulsations were 20 times larger than those of the synchronous component; whereas, in the horizontal axis turbine, the amplitudes of asynchronous pressure pulsations were two times smaller than those of the synchronous component.

In [9], the flow and vortex structure in the draft tube of Francis turbines at off-design condition were investigated using numerical simulation, and the comparative assessment was performed for RSM method and hybrid RANS/LES method. Considered is a case of part load (PL) at a flow rate of only 35% of the best efficiency point (BEP). As a result, both approaches showed the vortex rope and pressure pulsation spectra reasonably well.

The hybrid RANS/LES method gave similar predictions as the RSM, but resolving a wider range of scales.

In [12], the interaction between the guide vane and the impeller was investigated using numerical simulation. High head Francis runners are subject to pressure pulsation by interaction between guide vane and runner, to ensure safe operation of such turbines, it is important to be able to predict these pulsations. This paper investigated influence of turbulence modeling, wall friction, viscosity and mesh on the amplitude of pressure pulsations.

In [13], the low frequency pressure pulsation in the draft tube was predicted using DES simulation and Frozen Rotor method in the model turbine. The study shows the effect of flow structure on the frequency and intensity of non-stationary processes in the flow path. At low wicket gate opening, the flow in the draft tube is highly swirled, wide recirculation zone forms under the runner and vortex rope rotates around it. The vortex rope has several period of the rotation during the startup and induce intensive low-frequency pressure pulsations. Then, the vortex rope gradually destroys and pressure pulsations becomes non-periodical. At last, only weak straight vortex core remains under the runner and the pressure pulsations reaches minimal magnitude. Good agreement between the experimental and calculated data shows that the algorithm is suitable for the simulating of the transient processes in hydraulic turbines.

In [14], splitter blades have been employed in high head Francis turbines in order to improve performance as well as their unsteady characteristics. In this work two different splitter designs were analyzed, and the performance results were compared to the baseline runner with no splitter blades used. The amplitude of pressure pulsation caused by the precessing vortex rope as well as the related frequency was compared for two different operating conditions and from the results a relationship between the pulsation frequency and splitter blade geometry was observed. The results showed that the introduction of the splitter blades alters the frequency response in the draft tube due to a reduction of the residual swirl downstream of the runner. It was shown that small geometry modifications can affect the behavior of the runner at part load conditions, suggesting that an optimum design could be obtained minimizing the effects of the part load vortex rope.

The effect of air injection on the intensity of pressure pulsations was studied [15, 21]. The pressure pulsations were predicted by numerical simulation and measured by the experiments for high-head Francis turbines at different operating conditions, including free-load, full-load [16-19, 24, 25]. The pressure pulsations generated during the emergency shutdown or reject of load in a hydraulic turbine are experimentally investigated in [20, 22, 23].

Taken together, previous studies have shown that the studies on pressure pulsations in hydraulic turbines are mostly based on free-load or off-design conditions and not on the best efficiency point. The model experiments were used to evaluate the operation stability by measuring the pressure pulsations at the different locations of designed and manufactured turbine, whereas the prediction by numerical simulation were used at the design stage. In addition, the variable-speed operation, the employing of splitter blades and the air injection were used for reduction of pressure pulsation. The effect of lean angle of runner blade leading edge on the pressure pulsation is not investigated yet, so the study on it was carried out in this paper.

The paper investigates variation of the pressure pulsation with the change of the lean angle of the blade leading edge for Francis-130 turbine based on the numerical simulation and thereby proposes a suitable range of the lean angle.

2. Performance prediction of the turbines by numerical simulations

The parameters of the related turbine are as follows.

Net head: H=250m

Net flow rate: Q=23m^3/s

Speed: n=428.5r/min

Diameter: D=1.9m

Net power: N=50000kW

Number of blades: Z=14

2.1 Geometric modeling of the runner

Figure 1 shows the 3D geometric model of the runner with lean angle of −20° and 20°.

a) runner with lean angle of -20° b) runner with lean angle of 20°

Fig 1: Shape of the runner with lean angle of -20° and 20°

In order to investigate the effect of lean angle of the leading edge, three-dimensional fluid model of the runner was created, where the lean angles of the leading edges were -20°, -10°, -5°, 0°, 5°, 10°, 20°. To investigate the effect of only the lean angle of the blade leading edge, the leading edge in the meridian section, trailing edge of the blade and rigging angles of the leading and trailing edge are kept unchanged, thereby the flow port area in the trailing edge is constant. Figure 2 shows the projections of the runner in three directions.

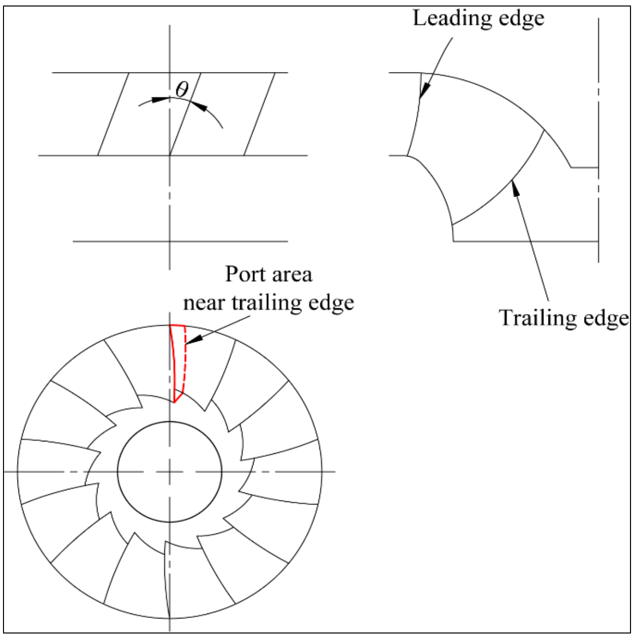


Fig 2: Projections of the runner

2.2 Performance evaluation of the turbine by numerical simulation

2.2.1 The computational domain and the grid

The 3D geometry for the spiral casing, stay vanes, guide vanes and draft tube region was modelled using SolidWorks, whose shape is shown in Figure 3.

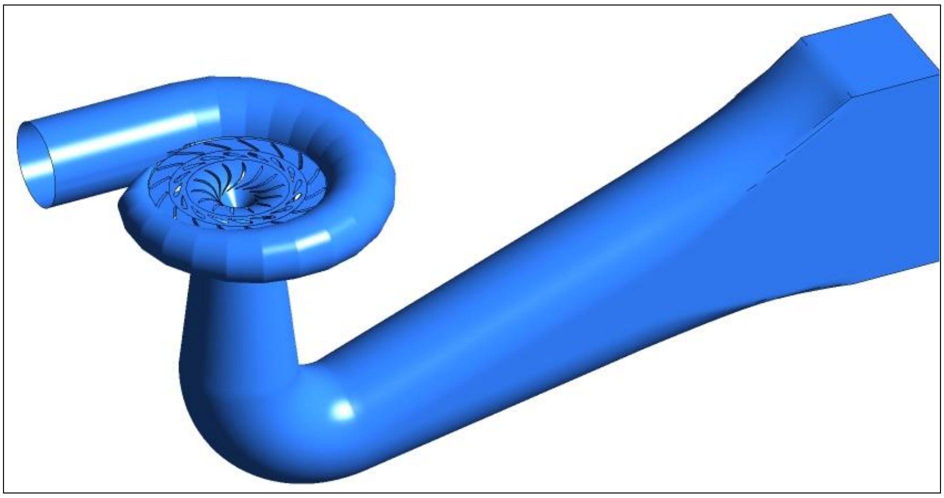
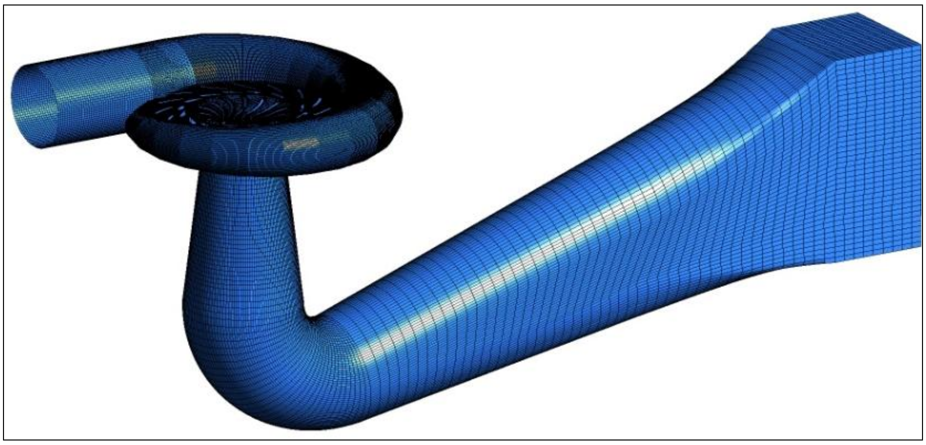
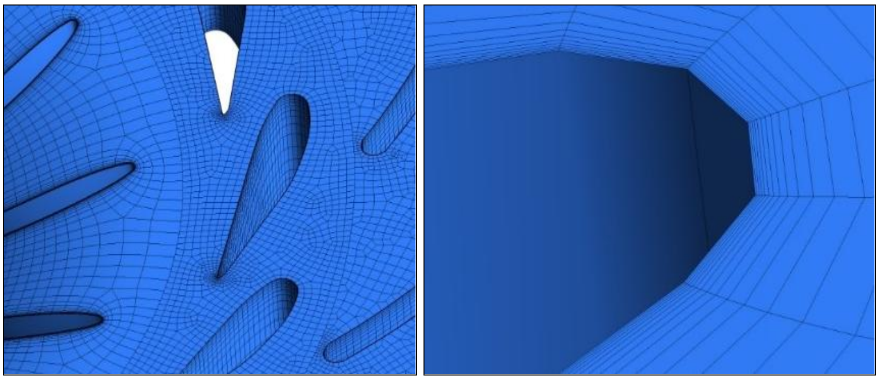


Fig 3: 3D model of Francis turbine passage

The grid for numerical simulation was created using the TurboGrid and ICEM-CFD tools. Figure 4 shows the mesh of the entire flow channel and the mesh near the guide vanes and runner blades.



a) Mesh of the entire flow passage



b) Mesh near the stay and guide vanes c) Mesh near the runner blades

Fig 4: Mesh of the computational domain

As shown in Figure 4, structured meshes are used in the domain of the spiral casing, stay vanes, guide vanes, runner and draft tube but unstructured meshed are used in the very small domain of the spiral. In addition, a boundary layer mesh with a thickness of 0.02mm of the first element layer was applied to all mesh near the wall because y^+ value is satisfied. The total number of elements in the entire domain is 23.6 million.

2.2.2 CFD calculation

The numerical simulations were carried out using CFD tool Fluent and SIMPLE scheme, pressure inlet and outlet condition were used for boundary conditions. While SST k–ω model was used for viscous model, schemes with high precision were used for discretization of all solution variables. Transient calculations were performed to compare the pressure pulsation characteristics and the pressure pulsations were obtained at 10 points inside the hydraulic turbine i.e. four points in the spiral casing, two points in the guide vane region, and four points in the draft tube. The locations of the measurement points are shown in Table 1 and Figure 5.

Table 1: Locations of the pressure pulsation measurement points

Points	X coordinate	Y coordinate	Z coordinate
P_{S1}	2.5	0	0
P_{S2}	−2.1	0	0
P_{S3}	0	−2.6	0
P_{S4}	0	2.3	0
P_{G1}	1.2	0.2	0
P_{G2}	0.2	−1.2	0
P_{D1}	0.82	0	0.9
P_{D2}	0.88	0	1.5
P_{D3}	0.94	0	2.2
P_{D4}	1.06	0	3.2

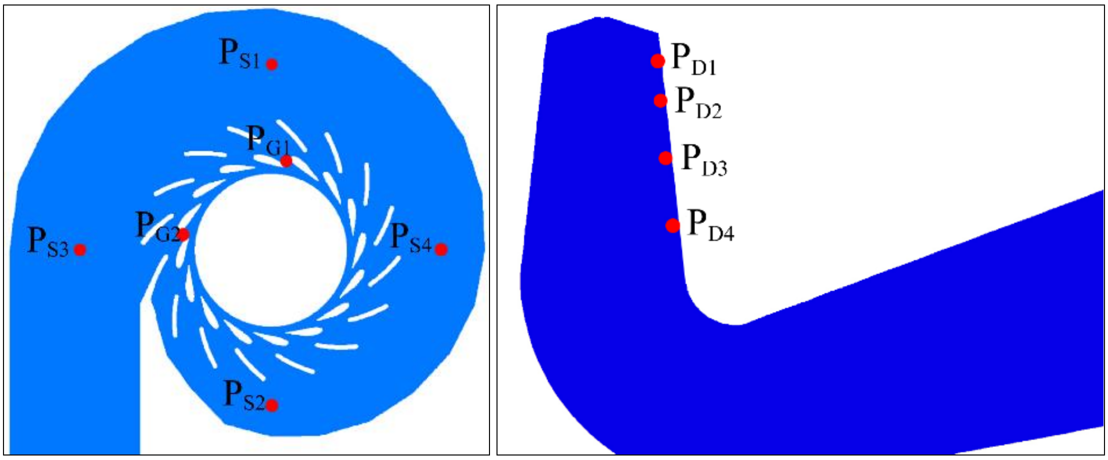


Fig 5: Locations of measurement points

In the transient calculation, the time step was set to 0.0001s considering the mesh size. The pressure pulsations obtained by transient calculations were analyzed through FFT transformation.

3. Results and analysis

3.1 The comparison of flow rate and efficiency

Recently, the hydro-turbine is usually operating at 12° of rotation of the guide vane. Hence, we compare the variation of the characteristics with the change of the lean angle of the runner blade leading edge at this operating conditions using CFD calculation.

Table 2 gives the flow rate, power and efficiency values for the lean angle of the runner blade leading edge at 12° of guide vane rotation angle, and Figure 6 shows the efficiency variation.

Table 2: The flow rates, power and efficiency values for the lean angle

Lean angle of leading edge, °	Flow rate, m^3/s	Power, kW	Efficiency,%
-20	14.33	31426	89.51
-10	14.35	31532	89.69
-5	14.33	31527	89.8
0	14.33	31464	89.62
5	14.31	31351	89.42
10	14.32	31332	89.31
20	14.32	31255	89.09

As shown in Table 2 and Figure 6, it can be seen that the flow rate tends to decrease with increasing lean angle, and the efficiency is highest when lean angle is -5°. In addition, it can also be seen that the flow rate variation is 0.28% and the efficiency variation is 0.71% when the lean angle of the runner blade leading edge is varied from -20° to -20°. Therefore, the effect of the lean angle change on the flow rate can be negligible but it can't be negligible for the efficiency.

3.2 Pressure distribution at the runner

Figure 7 shows the pressure distribution at the suction surface of the blade when the lean angle of the runner blade leading edge varies from -20° to -20°

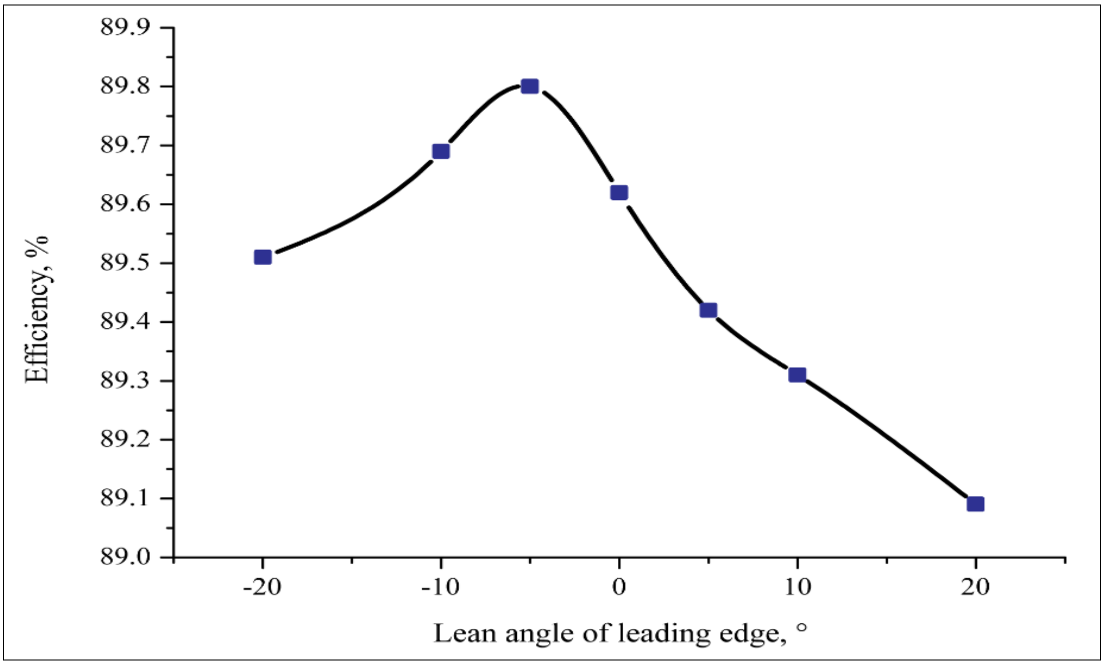
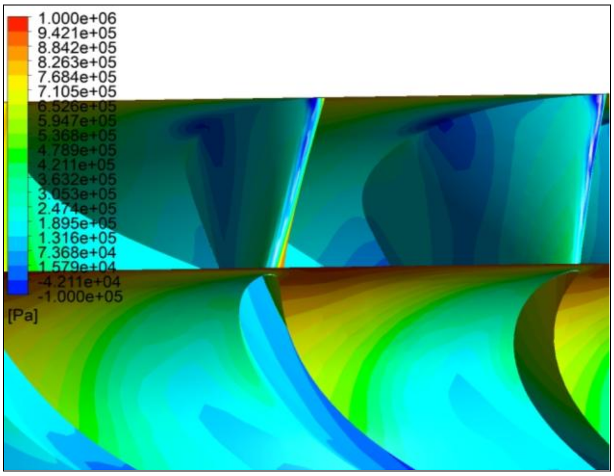
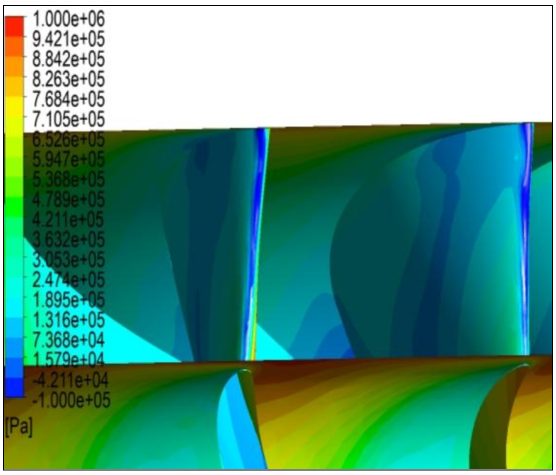


Fig 6: Efficiency curve with the variation of the blade lean angle

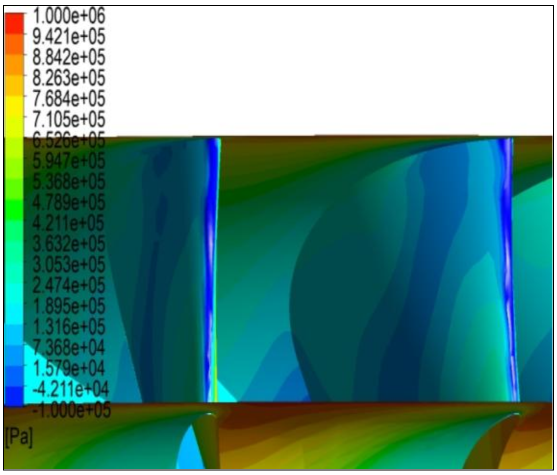
a) The position of the pressure distribution



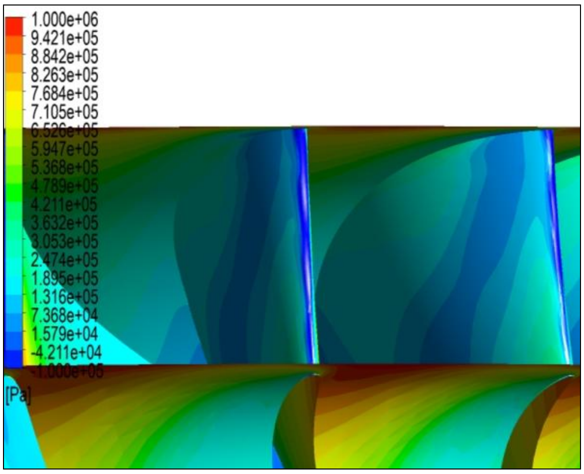
b) Runner with lean angle of -20°



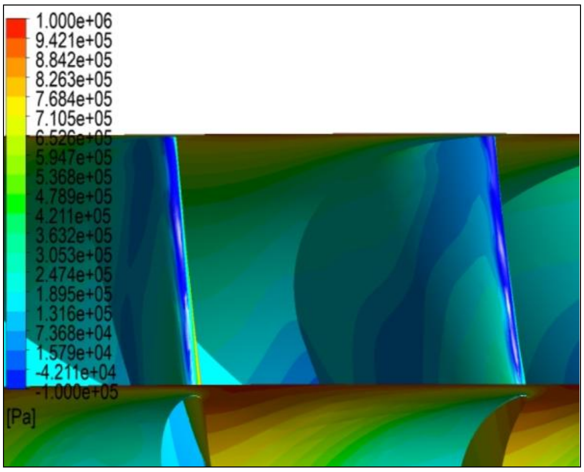
c) Runner with lean angle of -10°



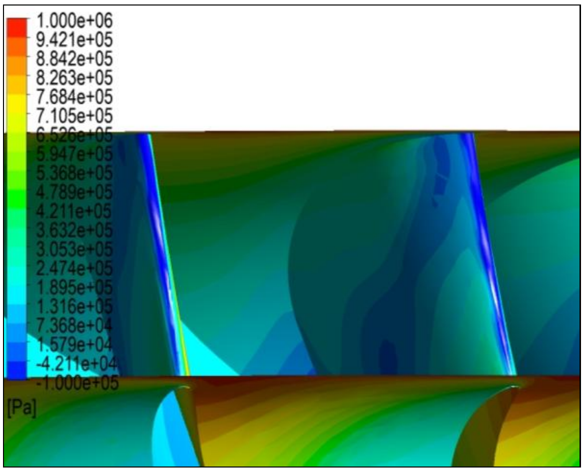
d) Runner with lean angle of -5°



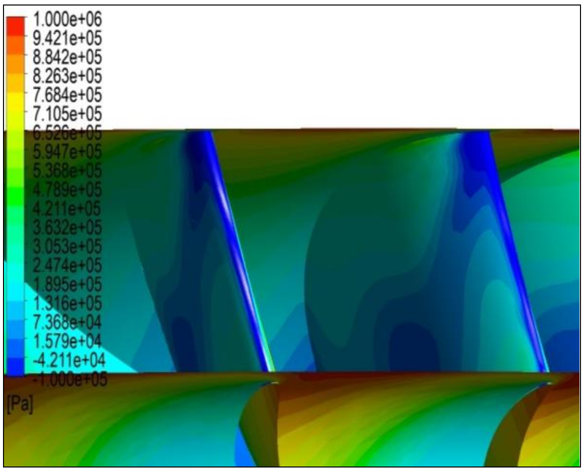
e) Runner with lean angle of 0°



f) Runner with lean angle of 5°



g) Runner with lean angle of 10°

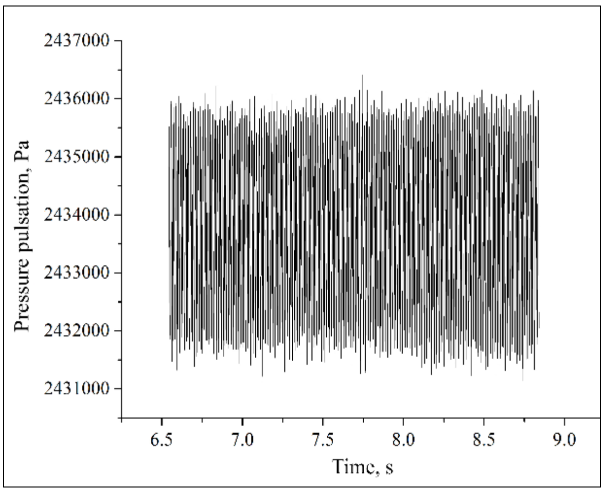


h) Runner with lean angle of 20°

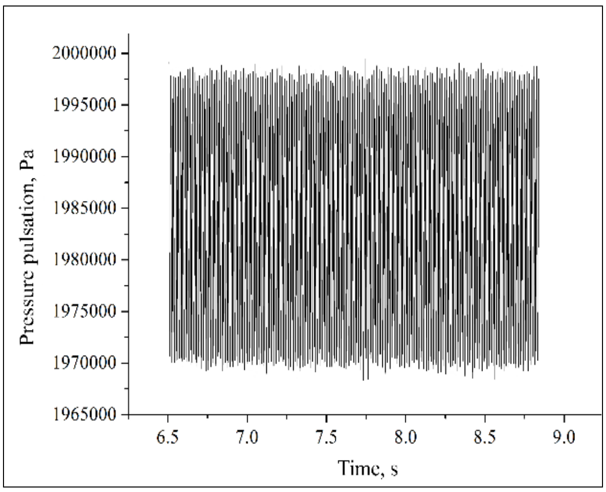
Fig 7: Pressure distribution in the suction surfaces

As shown in Figure 7, it can be seen that the region with negative pressure moves toward and downstream of the blade leading edge with increasing lean angle of leading edge.

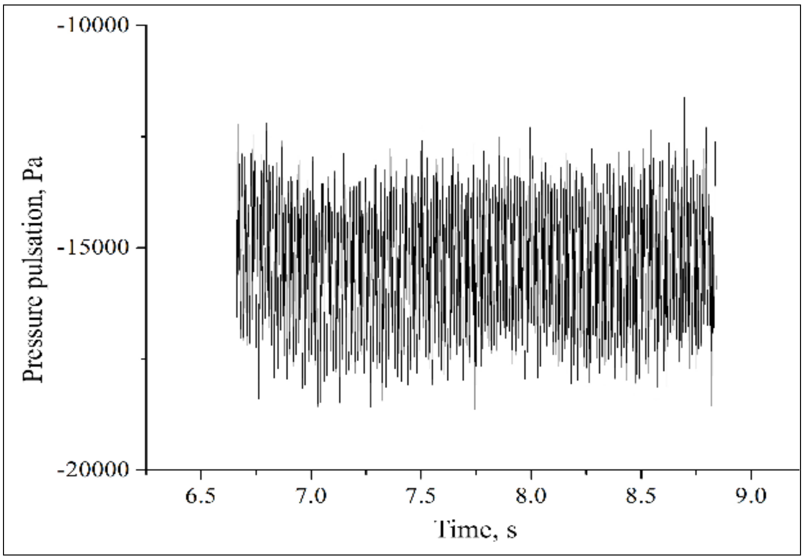
3.3. Pressure pulsations: The variation of pressure pulsation in the spiral casing, region between the guide vanes and the draft tube with the change of lean angle is analyzed. Figure 8 shows the pressure pulsation curves obtained at point P_{S1} in the spiral casing, point P_{G1} between the guides vane and point P_{D1} in the draft tube at 5° of lean angle.



a) Pressure pulsation at point P_{S1}



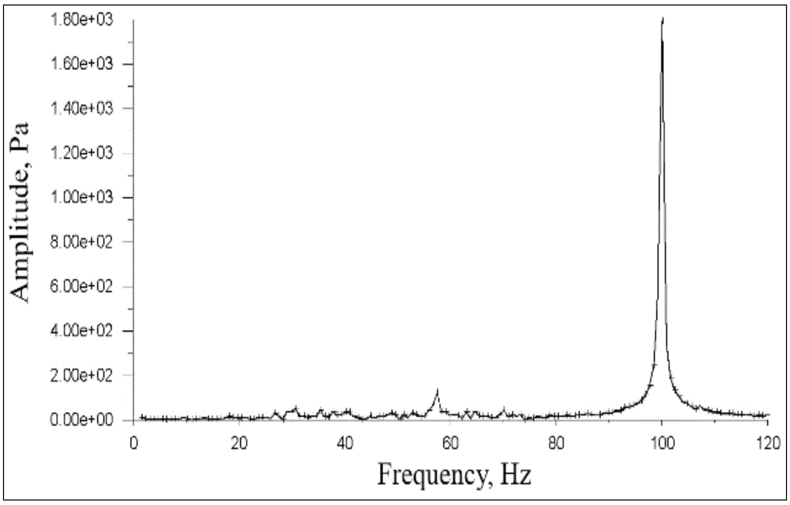
b) Pressure pulsation at point P_{G1}



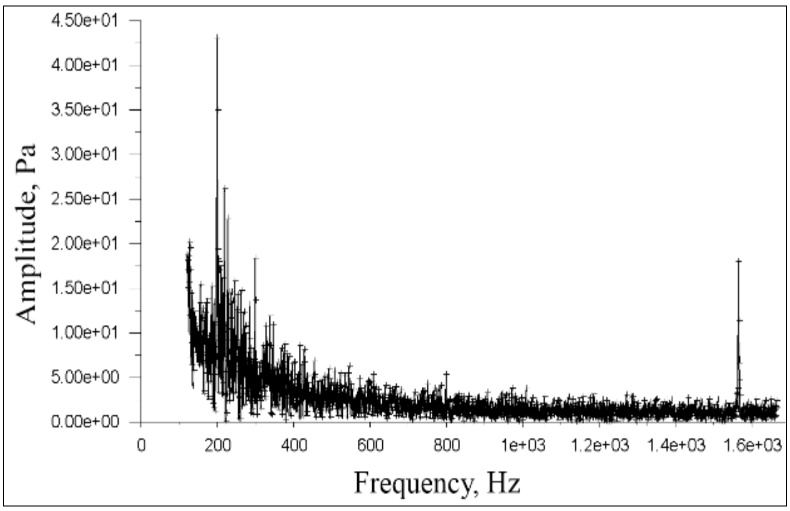
c) Pressure pulsation at point P_{D1}

Fig 8: Pressure pulsation curves

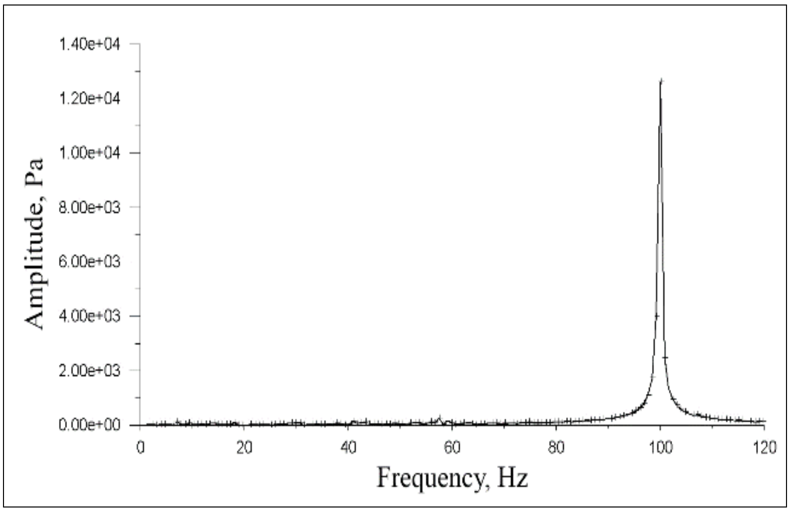
Since the runner speed is 428.5 rpm, the number of blades is 14, and the number of guide vanes 16, the pressure pulsations corresponding to f_1=n/60×1/3=2.3Hz, f_2=n/60×Z=100Hz, f_3=n/60×Z×Z_0=1600Hz are dominant. Hence, the pressure pulsation spectra in the range of 0~120 Hz and 120~1650 Hz is investigated using FFT technique. Figure 9 shows the spectra of pressure pulsations obtained at the point P_{S1} of the spiral casing, point P_{G1} between the guide vane and point P_{D1} of the draft tube at 10° of lean angle.



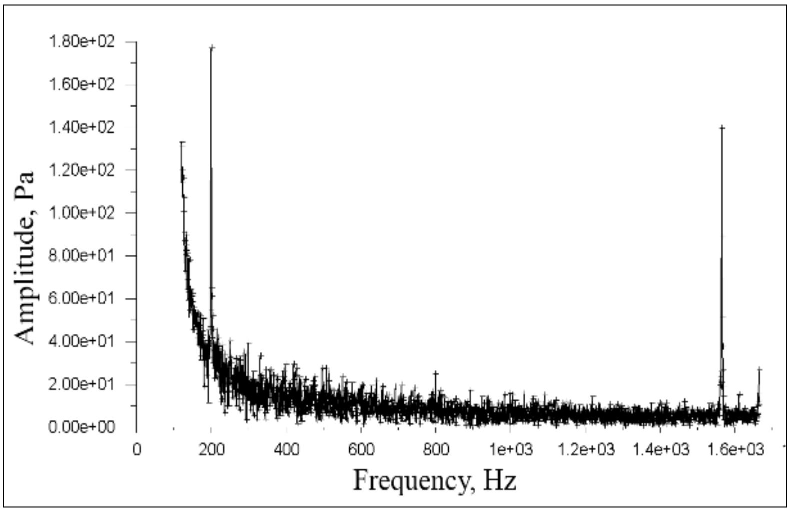
a) Point P_{S1} (in the range up to120Hz)



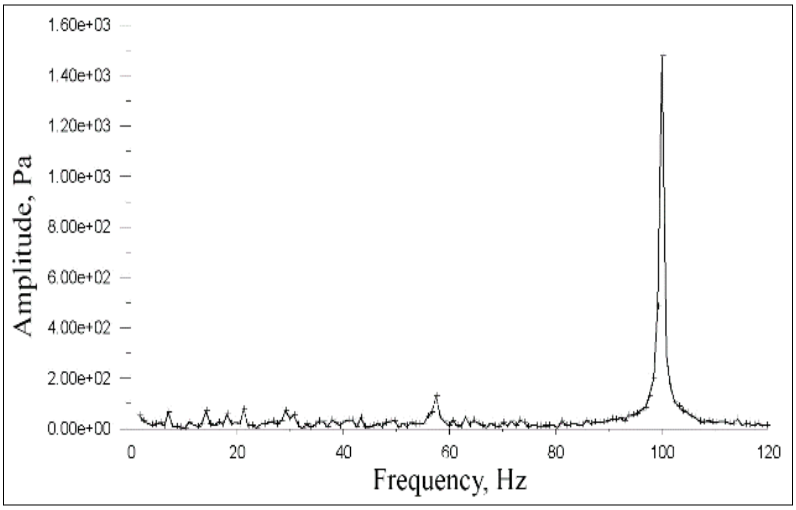
b) Point P_{S1} (in the range up to 1650Hz)



c) Point P_{G1} (in the range up to 120Hz)

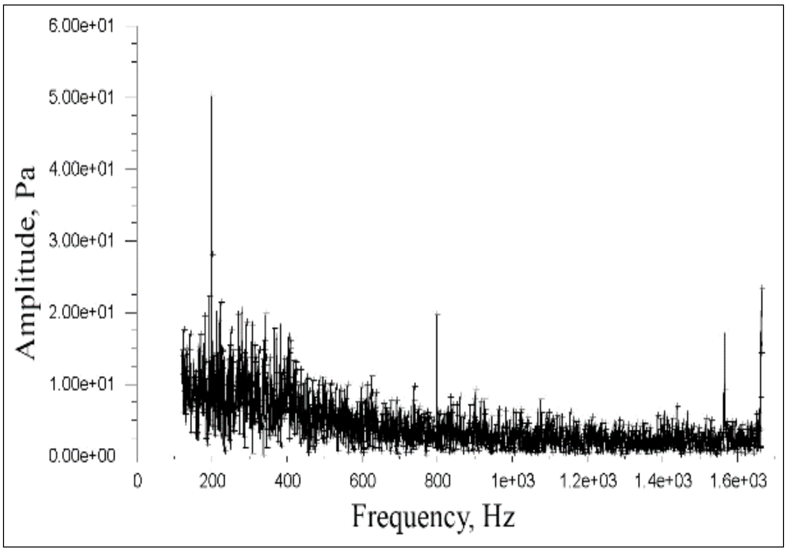


d) Point P_{G1} (in the range up to 1650Hz)



e) Point P_{D1} (in the range up to 120Hz)

As shown in Figure 9, it can be seen that the pressure pulsations at the points of interest of the spiral casing, guide vanes and draft tube all dominate the pulsations around 100 Hz. It can also be seen that the pressure pulsations around 200 and 1600 Hz are relatively significant, but they are very small compared to the pressure pulsations around 100 Hz. In general, the pressure pulsation frequency in the draft tube is about 1/3-1/4 compared to the runner rotation frequency, i.e., 1.8-2.3 Hz pulsation frequency. Considering the pressure pulsation spectrum in the draft tube, it can be seen that the pressure pulsation at this frequency does not occur. These pressure pulsations have the similar pattern for all lean angle of leading edge. Hence, only the pressure pulsation of 100 Hz is compared in this paper. Table 3 shows the pressure pulsation values of 100 Hz at the evaluating points



f) Point P_{D1} (in the range up to 1650 Hz)

Fig 9: Pressure pulsations in the spiral casing, guide vane and draft tube

Table 3: The pressure pulsation values of 100 Hz at the evaluating points.

LEAN Angle, °	Pulsation Frequency, Hz	P_{S1}, Pa	P_{S2}, Pa	P_{S3}, Pa	P_{S4}, Pa	P_{G1}, Pa	P_{G2}, Pa	P_{D1}, Pa	P_{D2}, Pa	P_{D3}, Pa	P_{D4}, Pa
-20	100	2186	6258	366	1246	12707	10186	1767	1535	1418	1278
-10	100	1940	6620	383	1230	13364	11195	1647	1540	1427	1276
-5	100	2013	6947	428	1340	14021	11610	1746	1660	1538	1379
0	100	1961	6371	359	1285	12818	10385	1687	1485	1376	1243
5	100	2034	6925	427	1301	13875	11171	1902	1686	1565	1414
10	100	1792	6101	403	1065	12619	10431	1478	1343	1242	1119
20	100	1533	5773	349	1046	10753	8970	1362	1315	1219	1100

Comparing the amplitude of pressure pulsations at different points, it can be found that the amplitude of pressure pulsations in the spiral casing are largest when the lean angle of the blade leading edge is -5° and smallest when it is 20°. The pressure pulsation in the spiral casing is relatively big near the end section (point P_{S2}), and the value varies by a maximum of 1170 Pa depending on the lean angle.

The pressure pulsation in the guide vane region is also the largest when the lean angle is -5° and the smallest when it is 20°. Here, the pressure pulsation with lean angle has a maximum difference of 3268 Pa at P_{G1} and a maximum difference of 2640 Pa at P_{G2}.

The pressure pulsation in the draft tube is the largest when the lean angle is 5°, and the value is almost similar to that of -5°. The pressure pulsation in the draft tube is the smallest when the lean angle is 20°. According to the lean angle of the blade leading edge, the pressure pulsation in the draft tube has a maximum of 540 Pa at point P_{D1}, a maximum of 371 Pa at point P_{D2}, a maximum of 346 Pa at point P_{D3}, and a maximum of 314 Pa at point P_{D4}.

From the comparison, it can be found that the amplitude of pressure pulsations in the spiral casing and the guide vane region are the largest when the lean angle is around -5°, and the amplitude of pressure pulsations in the draft tube are the largest when the lean angle is around 5°. The pressure pulsation at all evaluating points of the flow passage is the smallest when the lean angle is 20°. As a result, the lean angle of 20° is effective in terms of pressure pulsation for a Francis turbine with a relatively low specific speed.

Conclusion

The paper compares the efficiency of the hydraulic turbine, the pressure distribution in the runner and the amplitude of pressure pulsations in the flow channel when the lean angle of the runner blade leading edge varies from -20° to 20°. From the numerical simulations, it was found that the highest efficiency was 89.8% at the lean angle of -5° and the lowest efficiency was 89.09% at -20°.

As the lean angle increases, the region with negative pressure at the suction surface of the runner moves toward the leading edge. In other words, the region with negative pressure appears at the suction surface and crown sides when the lean angle decreases to -20°. Therefore, it can be concluded that the excessive reduction of the lean angle of the runner blade leading edge is unsuitable in terms of preventing cavitation.

The amplitude of pressure pulsations in the spiral casing, guide vane and draft tube region are relatively largest when the lean angle of the leading edge is -5°, and the small when 20°. So the higher lean angle is beneficial for pressure pulsation, while the performance is slightly degraded in efficiency.

From these results, it is concluded that the lean angle of the runner blade leading edge should be determined reasonably based on the comparative analysis of efficiency and pressure pulsations. The results of this study suggest that the lean angle of 20° for Francis-130 turbine is more suitable, because it is more important to increase the operating stability while the efficiency is less in the hydraulic turbine.

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Conflict of interests
The author declares no conflict of interest.

Disclosure statement
No potential conflict of interest was reported by the authors.

Data Availability
The data that support the findings of this study are available within the article.

References

1. Brekke H. *Hydraulic Turbines: Design, Erection and Operation*. Endringsdato. 2001;6:242-296.
2. Iliev I, Trivedi C. Variable-speed operation of *Francis turbines*: A review of the perspectives and challenges. Renewable and Sustainable Energy Reviews. 2019;103:109-121.
3. Wang S, Zhang L. Research on unsteady hydraulic features of a *Francis turbine* and a novel method for identifying pressure pulsation transmission path. 2019;6:1-17.
4. Platonov D, Minakov A. Numerical investigation of hydroacoustic pressure pulsations due to rotor-stator interaction in the *Francis-99 turbine*. Journal of Physics: Conference Series. 2019;1296:012009.
5. Favrel A, Pereira Junior JG. Dynamic modal analysis during reduced scale model tests of hydraulic turbines for hydro-acoustic characterization of cavitation flows. Mechanical Systems and Signal Processing. 2019;117:81-96.
6. Litvinov I, Shtork S. Unsteady regimes and pressure pulsations in draft tube of a model *hydro turbine* in a range of off-design conditions. Experimental Thermal and Fluid Science. 2018;91:410-422.
7. Sannes DB, Iliev I. Pressure pulsation in a high-head *Francis turbine* operating at variable speed. Journal of Physics: Conference Series. 2018;1042:012005.
8. Trivedi C, Gogstad PJ. Investigation of the unsteady pressure pulsations in the prototype *Francis turbines*-Part 1: Steady state operating conditions. Mechanical Systems and Signal Processing. 2018;108:188-202.
9. Gavrilov AA, Sentyabov AV. Vortical structures and pressure pulsations in draft tube of a *Francis-99 turbine* at part load: RANS and hybrid RANS/LES analysis. International Journal of Heat and Fluid Flow. 2017;63:158-171.
10. Litvinov I, Shtork S. Unsteady regimes and pressure pulsations in draft tube of a model *hydro turbine* in a range of off-design conditions. Experimental Thermal and Fluid Science. 2017;10:2-33.
11. Litvinov I, Gorelikov E. Regimes with periodical pressure pulsation in *Francis* draft tube. EDP Sciences. 2017;115:05013.
12. Østby PTK. On the relation between friction losses and pressure pulsations caused by rotor-stator interaction on the *Francis-99 turbine*. Journal of Physics: Conference Series. 2017;782:012010.
13. Minakov A, Sentyabov A, Platonov D. Numerical investigation of flow structure and pressure pulsation in the *Francis-99 turbine* during startup. Journal of Physics: Conference Series. 2017;782:012004.
14. Kassanos I, Anagnostopoulos J, Papantonis D. Numerical investigation of draft tube pressure pulsations in a *Francis turbine* with splitter blades. Journal of Physics: Conference Series. 2017;813:012049.
15. Qian ZD, Yang JD. Numerical simulation and analysis of pressure pulsation in *Francis hydraulic turbine* with air admission. National Natural Science Foundation of China. 2007;19(4):467-472.
16. Trivedi C, Cervantes MJ. Investigation of a high-head *Francis turbine* at runaway operating conditions. Energies. 2016;9(2):149-160.
17. Minakov AV, Platonov DV. The analysis of unsteady flow structure and low-frequency pressure pulsations in high-head *Francis turbines*. International Journal of Heat and Fluid Flow. 2015;53:183-194.
18. Minakov AV, Platonov DV. The numerical simulation of low-frequency pressure pulsations in high-head *Francis turbines*. Computers & Fluids. 2015;111:197-205.
19. Platonov D, Minakov A. Numerical and experimental study of low-frequency pressure pulsations in hydraulic units with *Francis turbine*. Journal of Physics: Conference Series. 2016;754:062007.
20. Trivedi C, Gandhi BK. Experimental investigations of a model *Francis turbine* during shutdown at synchronous speed. Renewable Energy. 2015;83:828-836.
21. Minakov A, Platonov D. Experimental study of the effect of air injection on the pressure pulsations in the *hydro turbine* flow path under different operating conditions. EDP Sciences. 2017;115:05001.
22. Trivedi C, Cervantes MJ. Transient pressure measurements on a high-head model *Francis turbine* during emergency shutdown, total load rejection, and runaway. Journal of Fluids Engineering. 2014;136(12):121107-1.
23. Zhang XX, Cheng YG. Simulation of the load rejection transient process of a *Francis turbine* by using a 1-D-3-D coupling approach. Journal of Hydrodynamics. 2014;26(5):715-724.
24. Shingai K, Okamoto N. Long-period pressure pulsation estimated in numerical simulations for excessive flow rate condition of *Francis turbine*. Journal of Fluids Engineering. 2014;136(7):071105-1.
25. Lipej A, Jošt D. Numerical prediction of pressure pulsation amplitude for different operating regimes of *Francis turbine* draft tubes. International Journal of Fluid Machinery and Systems. 2009;2(4):10-12.