E-ISSN: 2664-8784
P-ISSN: 2664-8776
Impact Factor: RJIF 8.26
IJRE 2025; 7(2): 124-136
© 2024 IJRE
www.engineeringpaper.net
Received: 14-08-2025
Accepted: 17-09-2025

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Selection method of grinding machine and air classifier in grinding-classification process by using FSFDMW-TOPSIS

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DOI: https://www.doi.org/10.33545/26648776.2025.v7.i2b.131

Abstract
In this paper, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) using the fuzzy score function of decision makers' weights (FSFDMW) in fuzzy set decision making problem is proposed to select the types of grinding machine and air classifier in dry grinding-classification process. Firstly, we consider how to select the main criteria among possible criteria that affect rational alternative decision making by using fuzzy score function with decision makers' weights, and propose a method to determine the weights of the main criteria by fuzzy score function with decision makers' weights. Then, we propose a TOPSIS method using fuzzy scoring functions with the decision makers' weights and choose a suitable types of grinding machine and air classifier for dental gypsum grinding.

Keywords: Decision makers weight, fuzzy score function, FSFDMW-TOPSIS method, grinding machine, air classifier, grinding-classification process

1. Introduction

In dry grinding-classification process, the type selection of grinding machine and air classifier is one of the most important issues in process design.

The optimal process design that meets the specifications of the grinding-classification process can be achieved by selecting the most suitable types of grinding machines and air classifiers for the grinding-classification process designed among the different grinding machine and air classifier types.

In order to select the appropriate grinding machine and air classifier types required in the process, first, the set of selected proposals and the criteria affecting selection must be selected, and the weight of the importance of those criteria must be determined. And also there must be a method to make a reasonable choice.

It should have the indefinite ambiguity how to select the criteria influenced to the types selection of reliable grinding machine and air classifier in grinding-classification process design, how to select the important criteria and how to select the reliable selection. And also no statistical data can be provided to resolve them. Therefore it must be solved by using the FSFDMW method based on the subjective opinion data of technician and experts in this field.

The fuzzy set decision making problem is to select the optimal proposal by integrating the expert opinion of the decision makers given the set of selected proposals, the set of different criteria to be considered in the selection and the set of decision makers for the case of uncertainty in the decision making environment.

One of the typical techniques for finding the most reasonable selection is Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). In the classical TOPSIS method, the decision making environment has no uncertainty (Wang 2006) [1].

To solve the decision making problem in the case of uncertainty in the decision making environment, a fuzzy TOPSIS method is proposed.

Chen (2000) proposed an extension of TOPSIS for group decision making under fuzzy environment [2].

Jahanshahloo *et al.* extended the concept of TOPSIS in developing a method for solving multi-criteria decision making problems with fuzzy data [3].

Applications of fuzzy group decision making problem using fuzzy TOPSIS have been extensively studied.

Chang developed a fuzzy TOPSIS method for optimal initial training aircraft evaluation [4].
Chu (2002) proposed a fuzzy TOPSIS model in a group decision making problem for solving aircraft trajectory location selection problem [5].
Gligoric *et al*. (2010) developed a model for arrow location selection in complex ore-body rocks as an optimal solution obtained by fuzzy TOPSIS and a useful location selection obtained by network optimization [6].
Doukas (2009) conducted a study on energy source renewal using fuzzy TOPSIS associated with carbon-gas emission reduction [7].
Fuzzy Hierarchical Analytics (AHP) is also widely used in group decision making problems.
Ghodsipour (2009) reported that fuzzy hierarchy analysis is one of the most comprehensive systems for multi-criteria decision making [8].
The classical fuzzy hierarchy analysis method is a method to perform consistency verification after making a one-to-one comparison questionnaire, making a decision matrix and making a decision by comparing each other two by two.
Safari (2010) used fuzzy hierarchical analysis (AHP) to search for mineral processing equipment considering eight criteria in iron ore mines [9].
Ataei (2013) applied the Monte Carlo hierarchical analysis method to select the most suitable underground mining method at Jashear bauxite mine [10].
Rahimdel (2014) used fuzzy hierarchy analysis to select the most suitable crusher among the major crushers available [11].
Karimnia (2015) determined the best mining method using fuzzy hierarchical analysis method in a salt mine in Iran [12].
In set decision making problems, studies combining fuzzy hierarchical analysis (AHP) and fuzzy TOPSIS have also been proposed, in which the weights of criteria are treated by fuzzy hierarchical analysis (AHP) and the rational selection is treated by fuzzy TOPSIS [13, 14].
Many researchers have introduced intuitionistic fuzzy set IFS with TOPSIS to provide a hybrid approach for multi-criteria decision making problems [15-16].
Many researchers have extended fuzzy TOPSIS for multi-criteria decision making on interval-valued intuitionistic fuzzy information and uncertain fuzzy information [18-21].
Recently, studies have also been proposed to solve uncertain multi-criteria problems in the case where the decision making environment is given as an uncertain fuzzy environment.
Xu and Zhang (2013) proposed a fuzzy multi-attribute decision making problem by TOPSIS with incomplete fuzzy information [22].
Li and Zhu (2015) proposed a TOPSIS method to solve uncertain multi-criteria decision making problems [23].
The TOPSIS method, AHP method, and their combination method, and decision-making method in uncertain fuzzy environment, studied in the previous section, were limited to one decision maker or did not consider the weight of the decision maker’s evaluation level in group decision-making.
The accuracy of fuzzy group decision making is strongly related to the limitations and the evaluation level of expert research on the problem domain of decision makers, and the number of decision makers.
Yue (2012) proposed an extension of TOPSIS for determining the weight of the decision makers for group decision making problems with uncertain information [17].
Zhang (2017) developed two nonlinear optimization models for various criteria group decision making problems with uncertain fuzzy information, one for minimizing divergence among individual uncertain fuzzy decision matrices and the other for minimizing divergence between each individual uncertain fuzzy matrix and the aggregate uncertain fuzzy matrix, and from these two explicit formulas the weights of decision makers and criteria are respectively derived [24].
However, the drawbacks of operations and methods in [24] are that in various criteria group decision making, the weights of decision makers are different due to the internal considerations and value personalities of decision makers according to each criterion, and the weights of criteria are not considered to depend on the number of decision makers, which increases the complexity of computation in practical applications.
In the decision making problem of determining a rational alternative among the various alternatives studied previously, the criteria affecting the alternatives are already known and there is no study that has proposed a method of selecting them.
To address this issue, in this paper, we first propose a method to select the main criteria influencing the selection by means of fuzzy equivalence clustering method using fuzzy score function with the weights of decision makers.
Next, we define an n-dimensional fuzzy environment with respect to the number of decision makers and develop a method to calculate the weights of decision makers for each criterion and the whole set of criteria by means of simple averaging.
Then, we develop a method to calculate the weights of criteria by a scoring function of each criterion with the decision makers’ weights for each criterion and a scoring function of the criteria with the decision makers’ weights for the whole set of criteria.
Next, we propose a TOPSIS decision-making method using fuzzy score function with the weights of decision makers and choose the appropriate type of grinding machine and air classifier for dental gypsum grinding.
The main advantage of this method over other currently available methods is that it is possible to increase the accuracy of decision making more realistically by calculating the weights of decision makers in group decision making, and to apply simple averaging operations.

2. Methodology

2.1 Main criteria selection method by using FSFDMW

In order to make a reasonable selection among the selection proposals, we have to choose the several criteria influenced to the selection.
Among the possible criteria affecting group decision making for rational selection, the basic and main criteria can be selected using multivariate analysis based on statistical data, but this method cannot be applied for qualitative criteria where statistical data cannot be obtained.
Therefore we propose a method to select the main criteria among the possible criteria to consider based on the knowledge of decision makers (technicians and experts).
The set of possible criteria affecting a rational selection is called $C = \{c_1, c_2, \cdots, c_m\}$, and the set of decision makers to choose the main criteria is called $Z = \{z_1, z_2, \cdots, z_n\}$.
The subjective opinion data of decision makers (technicians, experts) for selecting the main criteria is made as shown in Table 1.

Table 1: The subjective opinion data of decision makers for selecting the main criteria

C	*c*1	*c*2	…	*cj*	…	*cm*
Z_1	e_{11}	e_{12}	…	e_{1j}	…	e_{1m}
Z_2	e_{21}	e_{22}	…	e_{2j}	…	e_{2m}
⋮	⋮	⋮	…	⋮	…	⋮
Z_k	e_{k1}	e_{k2}	…	e_{kj}	…	e_{km}
⋮	⋮	⋮	…	⋮	…	⋮
Z_n	e_{n1}	e_{n2}	…	e_{nj}	…	e_{nm}

In the table 1, $e_{kj}(k=\overline{1,n}\,,\ j=\overline{1,m})$ is the values between [0, 1], and *k*-th decision maker denots his subjective opinion as the value between [0, 1] the importance degree of the *j*-th criterion c_j .

Obviously, we give the value 0 for the secondary criterion, and clearly the value 1 for the main criterion, and the value between [0, 1] according to its importance degree.

Definition 1: We can define *k*-th decision maker's weight for *j*-th criterion c_j as follows:

$$w'_{kj}=1-\left|(\frac{1}{n}\sum_{l=1}^{n}e_{lj})-e_{lj}\right|,\quad (k=1,2,\cdots,n,\ \ j=1,2,\cdots,m) \quad (1)$$

When the eq.(1) is standardized, the *k*-th decision maker's weight for *j*-th criterion c_j is defined as follows.

$$w_{kj}=\frac{w'_{kj}}{\sum_{l=1}^{n}w'_{lj}}\,,\ w_{k\,j}\geq 0\,,\ \sum_{k=1}^{n}w_{k\,j}=1\,(j=1,2,\cdots,m) \quad (2)$$

Definition 2: We can define the fuzzy score function values with decision maker's weight for *j*-th criterion $c_j\ (j=1,2,\cdots,m)$ as follows:

$$S(x_j)=\frac{1}{n}\sum_{k=1}^{n}w_{k\,j}e_{kj}\ ,\ (j=1,2,\cdots,m) \quad (3)$$

The algorithm for selecting the main criteria is as follows.

Step 1: The subjective opinion data of decision makers (technicians and experts) is made for selecting the main criteria.

Step 2: By using the eq. (1) and (2), it calculates the decision maker's weight.

Step 3: It calculates the fuzzy score function values with decision maker's weight by using the eq. (3).

Step 4: We can define the 2-demention fuzzy equivalence relation $\underset{\sim}{R}{}_{C}$ as follows;

$$\underset{\sim}{R}{}_{C}\left(x_\upsilon,x_\omega\right)=\begin{cases}1 & ,\quad \upsilon=\omega \\ \min\{\bar{e}_\upsilon,\bar{e}_\omega\} & ,\quad \upsilon\neq\omega\end{cases}\ \left(\upsilon,\omega=\overline{1,m}\right) \quad (4)$$

where

$$\bar{e}_u=S(x_u)=\frac{1}{n}\sum_{k=1}^{n}w_{ku}e_{ku} \quad (5)$$

It is the fuzzy score function values with decision maker's weight for the criterion $c_u\ (u=1,2,\cdots,m)$

Step 5: We can define the fuzzy equivalence classification level $\lambda\in[0,1]$ with realistic means.

Step 6: We can determine the *α-cut* matrix $R_{C\lambda}$.

$$R_{C\lambda}\left(x_\upsilon,x_\omega\right)=\begin{cases}1 & ,\quad \underset{\sim}{R}{}_{C}\left(x_\upsilon,x_\omega\right)\geq\lambda \\ 0 & ,\quad \underset{\sim}{R}{}_{C}\left(x_\upsilon,x_\omega\right)<\lambda\end{cases} \quad (6)$$

Step 7: We can calculate the maximum value of fuzzy score function values with decision maker's weight for the criterion $c_\tau\ (\tau=1,2,\cdots,m)$ in fuzzy equivalence relation matrix $\underset{\sim}{R}{}_{C}$.

$$\bar{e}_\eta=\max_{1\leq\tau\leq m}\{\bar{e}_\tau\} \quad (7)$$

Step 8: We can determine the fuzzy equivalence classification by using the *α-cut* matrix.

If $R_{C\alpha}\left(x_\upsilon,x_\omega\right)=R_{C\alpha}\left(x_\omega,x_\upsilon\right)=1$ in $R_{C\alpha}$, criterion c_υ and criterion c_ω belong to the same fuzzy equivalence class.

The criteria belonging to the fuzzy equivalence class such as the η-th criterion c_η by eq. (7) are chosen as the main criteria that have a decisive influence on the *α-cut* matrix.

The *α-cut* matrix can be defined to reflect the actual requirement.

2.2 Main criteria weight determining method by using FSFDMW

In this section, we propose a fuzzy group decision making method to determine the weights of various criteria if the fuzzy environment is given as an n-dimensional fuzzy environment by n decision makers.

Because of the different experience and characteristics of decision makers in real environment, the weights of decision makers for each criterion are different. So, the weight for the set of whole criteria is related to the weight of decision makers for each criterion.

Therefore, considering the different weights of decision makers for each criterion, determining the weights of decision makers for the set of whole criteria will be more accurate in approaching the real problem.

To address this issue, we assume the following:

- The importance degree of criteria varies according to the selection.
- The degree of evaluation levels of decision makers on the set of whole criteria depends on the degree of evaluation levels of decision makers on each criterion.
- The number of weights of criteria for a selection depends on the number of decision makers.

For a set $C=\{c_1,c_2,\cdots,c_m\}$ of criteria, the set of decision makers to determine the weight of importance degree for each criterion is called $Z=\{z_1,z_2,\cdots,z_n\}$.

When $h_j=\{h_{j1},h_{j2},\cdots,h_{jn}\},\ h_{jk}\in[0,1],\ (j=1,\cdots,m)$ is the weight of importance degree for *j*-th criterion of n decision makers on the set of criteria *C*, h_j is an element of the n-dimensional fuzzy environment and $H=\{h_1,h_2,\cdots,h_m\}$ is the set of all elements in the n-dimensional fuzzy environment on the set of criteria *C*.

Definition 3: Given a map h_{E_n} that maps a set $C=\{c_1,c_2,\cdots,c_m\}$ of criteria to a subset of n-dimensional values of [0, 1] by a set $Z=\{z_1,z_2,\cdots,z_n\}$ of decision makers, we say that the map h_{E_n} determines an n-dimensional fuzzy set E_n on the set of criteria C, and $h_{E_n}(c_j),\ c_j\in C(j=1,2,\cdots,m)$ is called the membership function of an n-dimensional fuzzy set E_n.

To be easily understood, the n-dimensional fuzzy set (n-DFS) is expressed by mathematical symbols as follows:

$$E_n=(<c_j,h_{E_n}(c_j)>|c_j\in C,\ j=1,2,\cdots,m) \tag{8}$$

Where $h_{E_n}(c_j)$ is a subset of some n-dimensional values in [0, 1] and denotes the degree of probability that element $c_j\in C$ belongs to n-dimensional fuzzy set E_n.

For convenience, we call $h_j=h_{E_n}(c_j)$, $(j=1,2,\cdots,m)$ an element of n-dimensional fuzzy set for the *j*-th criterion c_j, and H_n a set of elements of all n-dimensional fuzzy sets.

Let us assume that element $h_j=\{h_{j_1},h_{j_2},\cdots,h_{j_n}\},\ h_{j_k}\in[0,1]\ (j=1,2,\cdots,m,\ \ k=1,\cdots,n)$ of a n-DFS for criterion c_j permitting duplication of h_{jk} on the set of criteria *C* was given.

Where h_{jk} is related to an intention of *k*-th decision maker for *j*-th criterion c_j.

We can define *k*-th decision maker's weight for element h_j of n-DFS for *j*-th criterion c_j as follows:

$$w'_{jk}=1-\left|\left(\frac{1}{n}\sum_{l=1}^{n}h_{jl}\right)-h_{jk}\right|,\quad (k=1,2,\cdots,n,\ \ j=1,2,\cdots,m) \tag{9}$$

where $w'_{jk}\ (j=1,2,\cdots,m)$ is the weight of evaluation level degree of the *k*-th decision maker in h_j.

By considering the conditions $w_{jk}\ge 0$, $\sum_{k=1}^{n}w_{jk}=1\ (j=1,2,\cdots,m)$, *k*-th decision maker's weight w_{jk} to element h_j of a n-DFS for *j*-th criterion c_j is determined as:

$$w_{jk}=\frac{w'_{jk}}{\sum_{p=1}^{n}w'_{jp}},\ (k=1,2,\cdots,n,\ j=1,2,\cdots,m)$$

If the evaluation level degrees of the decision makers are all the same, that is, $h_{j1}=h_{j2}=\cdots=h_{jn}$, then

$$w_{jk}=\frac{1}{n}\ (k=1,2,\cdots,n).$$

Definition 4: The fuzzy score function with the weights to h_j for *j*-th crterion c_j is defined as follows:

$$S(h_j)=\frac{1}{n}\sum_{k=1}^{n}w_{j_k}h_{jk}\ ,\ (j=1,2,\cdots,m) \tag{10}$$

If the evaluation level degrees of the decision makers are all the same, that is, if $h_{j1}=h_{j2}=\cdots h_{jn}=h'_j$, then $S(h_j)=h'_j$.

We can define the weights of evaluation level degrees of decision makers for H_n to set $H_n=\{h_1,h_2,\cdots h_m\}$ of elements of all n-dimensional fuzzy set (n-DFS) as follows:

$$w'_k=1-\frac{1}{m}\left[\sum_{j=1}^{m}\left|S(h_j)-h_{jk}\right|\right],(k=1,2,\cdots,n) \tag{11}$$

Determining the weight w_k of evaluation level degree of decision maker for H_n to be $w_k\ge 0$ and $\sum_{k=1}^{n}w_k=1$, then

$$w_k=\frac{w'_k}{\sum_{p=1}^{n}w'_p},(k=1,2,\cdots,n).$$

Definition 5: Let us assume that the set $H_n=\{h_1,h_2,\cdots,h_m\}$ of all n-DFS elements on the set of criteria $C=\{c_1,c_2,\cdots,c_m\}$ was given.

We define the fuzzy score function with weights for criteria $c_j\ (j=1,2,\cdots,m)$ on H_n as following:

$$S_j(H_n)=\frac{1}{n}\sum_{k=1}^{n}w_k h_{jk}\ ,(j=1,2,\cdots,m) \tag{12}$$

Then we can determine the weights $W_{c_j}\ \ (j=1,2,\cdots,m)$ of importance degree to each criterion on criteria set $C=\{c_1,c_2,\cdots,c_m\}$ as follow:

$$W_{c_j}=\frac{S_j(H_n)}{\sum_{l=1}^{m}S_l(H_n)},\ (j=1,2,\cdots,m),\ W_{c_j}\ge 0\ \sum_{j=1}^{m}W_{c_j}=1 \quad (13)$$

2.3. TOPSIS method by using FSFDMW

In this section, we propose the TOPSIS method by using the fuzzy score function with decision maker's weight.

Let $V=\{V_1,V_2,\cdots,V_T\}$ be the set of selections to determine a rational selection proposal, $C=\{c_1,c_2,\cdots,c_m\}$ be the set of criteria affecting the alternatives, and $Z=\{z_1,z_2,\cdots,z_n\}$ be the set of decision makers to determine a rational selection proposal.

Dominance evaluation opinion data of n decision makers for *j*-th criterion c_j on *T* selections is given as shown in table 2.

Table 2: Dominance evaluation opinion data of n decision makers for *j*-th criterion c_j on *T* selections

	V_1	V_2	…	V_t	…	V_T
Z_1	h_{1j1}	h_{2j1}	…	h_{tj1}	…	h_{Tj1}
Z_2	h_{1j2}	h_{2j2}	…	h_{tj2}	…	h_{Tj2}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Z_k	h_{1jk}	h_{2jk}	…	h_{tjk}	…	h_{Tjk}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Z_n	h_{1jn}	h_{2jn}	…	h_{tjn}	…	h_{Tjn}

In table 2, we can define the weights of *k*-th decision maker for n-DFS elements $h_{tj}\ (t=1,2,\cdots,T)$ of *t*-th selection $V_t\ (t=1,2,\cdots,T)$ on *j*-th criterion c_j as follows:

$$w'_{tjk}=1-\left|\left(\frac{1}{n}\sum_{l=1}^{n}h_{tjl}\right)-h_{tjk}\right|,\quad (k=1,2,\cdots,n,\ t=1,2,\cdots,T) \quad (14)$$

where $w'_{tj\,k}\ (t=1,2,\cdots,T)$ is the weight of evaluation level degree of the *k*-th decision maker to h_{tj}.

By considering the conditions $w_{tj_k}\ge 0$, $\sum_{k=1}^{n}w_{tj_k}=1\ (t=1,2,\cdots,T)$, *k*-th decision maker's weight w_{tjk} to element h_{tj} of a n-DFS for *j*-th criterion c_j is determined as follows:

$$w_{tjk}=\frac{w'_{tjk}}{\sum_{p=1}^{n}w'_{tjp}},\ (k=1,2,\cdots,n,\ t=1,2,\cdots,T)$$

If the evaluation level degrees of the decision makers are all the same, that is, if $h_{tj1}=h_{tj2}=\cdots=h_{tjn}$, then $w_{tjk}=\frac{1}{n}\ (k=1,2,\cdots,n)$.

Definition 6: The fuzzy score function with the weights of n-DFS elements $h_{tj}\ (t=1,2,\cdots,T)$ to *t*-th selection $V_t\ (t=1,2,\cdots,T)$ for *j*-th criterion c_j is defined as follows:

$$S(h_{tj})=\frac{1}{n}\sum_{k=1}^{n}w_{tj\,k}h_{tjk},\ (t=1,2,\cdots,T) \quad (15)$$

If the evaluation level degrees of the decision makers are all the same, that is, if $h_{tj1}=h_{tj2}=\cdots h_{tjn}=h'_{tj}$, then $S(h_{tj})=h'_{tj}$.

Then we can define the weights of evaluation level degrees of decision makers to the set $H_{nj}=\{h_{1j},h_{2j},\cdots,h_{Tj}\}$ of all n-DFS elements on the set *V* of selections as follow:

$$w'_{jk}=1-\frac{1}{T}\left[\sum_{t=1}^{T}\left|S(h_{tj})-h_{tjk}\right|\right],(k=1,2,\cdots,n) \quad (16)$$

By considering the conditions $w_{tj_k}\ge 0$, $\sum_{k=1}^{n}w_{tj_k}=1\ (t=1,2,\cdots,T)$, *k*-th decision maker's weight w_{jk} to element H_{nj} is determined as follows:

$$w_{jk}=\frac{w'_{jk}}{\sum_{p=1}^{n}w'_{jp}},(k=1,2,\cdots,n)$$

Definition 7: Let us assume that the set $H_{nj}=\{h_{1j},h_{2j},\cdots,h_{Tj}\}$ of all n-DFS elements on the set $V=\{V_1,V_2,\cdots,V_T\}$ of selections was given.

We define the fuzzy score function with weights of decision makers to selections $V_t\ (t=1,2,\cdots,T)$ on H_{nj} as following:

$$S'_{tj}(H_{nj})=\frac{1}{n}\sum_{k=1}^{n}w_{tk}h_{tjk},(t=1,2,\cdots,T) \quad (17)$$

Normalized, the fuzzy score function with weights of decision makers to selection $V_t\ (t=1,2,\cdots,T)$ on H_{nj} is determined as following:

$$S_{tj}(H_{nj})=\frac{S'_{tj}(H_{nj})}{\sum_{t=1}^{T}S'_{tj}(H_{nj})},\ (j=1,2,\cdots,m),\ S_{tj}(H_{nj})\ge 0,\ \sum_{t=1}^{T}S_{tj}(H_{nj})=1 \quad (18)$$

From eq. (18), the fuzzy score function value matrix with weights of decision makers is given in Table 3.

Table 3: Fuzzy score function value matrix with weights of decision makers

	c_1	c_2	…	c_j	…	c_m
V_1	$S_{11}(H_{n1})$	$S_{12}(H_{n2})$	…	$S_{1j}(H_{nj})$	…	$S_{1m}(H_{nm})$
V_2	$S_{21}(H_{n1})$	$S_{22}(H_{n2})$	…	$S_{2j}(H_{nj})$	…	$S_{2m}(H_{nm})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_t	$S_{t1}(H_{n1})$	$S_{t2}(H_{n2})$	…	$S_{tj}(H_{nj})$	…	$S_{tm}(H_{nm})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_T	$S_{T1}(H_{n1})$	$S_{T2}(H_{n2})$	…	$S_{Tj}(H_{nj})$	…	$S_{Tm}(H_{nm})$

The algorithm of TOPSIS method by using the fuzzy score function with weights of decision makers to determine a rational selection proposal is as follows.

Step 1: For the *j*-th criterion c_j, we construct the dominance evaluation opinion data of n decision makers for T selections. Such data are generated for each of the m criteria.

Step 2: By using eq.(14) to the data constructed in step 1 for each criterion $c_j\ (j=1,2,\cdots,m)$ of m criteria, the weights of *k*-th decision makers for $h_{tj}\ (t=1,2,\cdots,T,\ j=1,2,\cdots,m)$ are calculated and normalized.

Step 3: By using eq.(15) to the data constructed in step 1 for each criterion $c_j\ (j=1,2,\cdots,m)$ of m criteria, we calculate the fuzzy score function with weights of n-DFS elements $h_{tj}\ (t=1,2,\cdots,T,\ j=1,2,\cdots,m)$ to *t*-th selection $V_t\,(t=1,2,\cdots,T)$.

Step 4: By using eq.(16), we calculate and normalize the weights of evaluation level degrees of decision makers to $H_{nj}=\{h_{1j},h_{2j},\cdots,h_{Tj}\}(j=1,2,\cdots,m)$ for each criterion $c_j\ (j=1,2,\cdots,m)$ of m criteria.

Step 5: By using eq.(17), we calculate the fuzzy score function with weights for selections $V_t\,(t=1,2,\cdots,T)$ on $H_{nj}=\{h_{1j},h_{2j},\cdots,h_{Tj}\}(j=1,2,\cdots,m)$ and then calculate eq.(18).

Step 6: By using eq.(19), the decision making matrix with the fuzzy score function values and the weights of criteria is given in Table 4.

$$y_{tj}=W_{c_j}S_{tj}(H_{nj})(t=1,2,\cdots,T,\ \ j=1,2,\cdots,m) \qquad (19)$$

Table 4: Decision making matrix

	c_1	c_2	…	c_j	…	c_m
V_1	y_{11}	y_{12}	…	y_{1j}	…	y_{1m}
V_2	y_{21}	y_{22}	…	y_{2j}	…	y_{2m}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_t	y_{t1}	y_{t2}	…	y_{tj}	…	y_{tm}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_T	y_{T1}	y_{T2}	…	y_{Tj}	…	y_{Tm}

Step 7: In the decision making matrix, the positive ideal solution (PIS) and the negative ideal solution (NIS) are determined as follows:

$$\begin{cases} A^+=(y_1^+,y_2^+,\cdots y_m^+) \\ A^-=(y_1^-,y_2^-,\cdots y_m^-) \end{cases} \qquad (20)$$

$$\begin{cases} y_j^+=\{(\max_{1\le t\le T} y_{tj}\mid j\in J_1),(\min_{1\le t\le T} y_{tj}\mid j\in J_2)\} \quad (j=1,2,\cdots,m) \\ y_j^-=\{(\min_{1\le t\le T} y_{tj}\mid j\in J_1),(\max_{1\le t\le T} y_{tj}\mid j\in J_2)\} \quad (j=1,2,\cdots,m) \end{cases} \qquad (21)$$

where J_1 is the index set of the beneficiary criteria and J_2 is the index set of the losing criteria.
Step 8: By means of positive and negative ideal solutions, the distance between the dominance evaluation value for the selection is calculated as follows:

$$D_t^+=\sqrt{\sum_{j=1}^{m}\left(y_{tj}-y_j^{\ +}\right)^2} \quad (t=1,2,\cdots,T) \qquad (22)$$

$$D_t^-=\sqrt{\sum_{j=1}^{m}\left(y_{tj}-y_j^-\right)^2} \quad (t=1,2,\cdots,T) \qquad (23)$$

Step 9: The priority value of each selection is calculated as follows:

$$B_t=\frac{D_t^-}{D_t^+ + D_t^-} \quad (t=1,2,\cdots,T) \qquad (24)$$

Step 1: Determine the ranking of selections.
The best selection for column B_t is the one with the smallest distance in the positive ideal solution and the one with the greatest distance in the negative ideal solution.

3. Type selection of grinding machine and air classifier in dental gypsum grinding-classification process

3.1 Type selection of grinding machine in dental gypsum grinding-classification process

With the rapid development of science and technology, the demand for new materials is increasing, especially with the strict size distribution of the particles of various powder materials, the specific particle shape, and the extremely low inclusion rate of impurities.
Therefore, grinding machines based on different grinding principles have been developed and applied widely in industry, and their structure and performance are being further improved.
The grinding machines currently used in the grinding industry are of great variety and type, and the performance of grinding machines is also variable.
Generally, the technical and economic efficiency of a grinding-classification process will also vary depending on the type of grinding machine selected in the grinding-classification process.
Therefore, the selection of the type of grinding machine that is most suitable for practical conditions is one of the important requirements for improving the technical and economic efficiency of the grinding-classification process.
We consider the appropriate type selection method of the grinding machine to establish the dental gypsum grinding-classification process based on the previous methodology.
Currently, there are the following types of grinding machines that are widely used in dry grinding process in the world:
Tumbling ball mill (A), centrifugal roller mill (B), vibrating mill (C), air-flow mill (D), planetary mill (E), stirred mill (F), impact mill (G), high pressure roller mill (H).
We choose the type of a suitable grinding mill for establishing the dental gypsum grinding-classification process from eight such mill types applied in dry grinding processes by using fuzzy group decision making methods.

3.1.1 Setting the main criteria for selecting type of grinding mill

In this paper, 10 technicians or/and experts are made up as decision makers. Possible criteria to consider in the selection of a suitable mill type are the fineness of the mill's product, the content of impurities contained in the mill's product, the capacity of mill, the mill's technical efficiency, the mill's specific power consumption, the mill's reliability, the manufacturing possibility of mill, the mill's manufacturing cost, the continuity of grinding working, the easy of repair in operation, the suitability of mill operation, and the mill's life.

The decision makers' opinion data for the selection of the main criteria are listed in Table 5.

First, the weights of the decision makers are calculated by using eq. (1) and eq. (2), and the fuzzy score function values with the weights of the decision makers are calculated by using eq. (3).

Then the binary fuzzy equivalence relation $\underset{\sim}{R}_C$ with $X = \{x_1, x_2, \cdots, x_{12}\}$ as the object space is calculated by using eq. (4) and eq. (5) in Table 5, and is represented as Table 6.

Table 5: Opinion data of decision makers for selecting the main criteria

	Fineness	**Content of impurities**	**Capacity**	**Technical Efficiency**	**Specific power consumption**	**Reliability**	**Manufacturing Possibility**	**Manufacturing cost**	**Continuity of working**	**Easy of repair**	**Suitability of operation**	**Life of mill**
1	0.9	0.9	0.7	0.3	0.5	0.7	0.3	0.3	0.3	0.4	0.4	0.4
2	0.9	0.8	0.9	0.4	0.5	0.8	0.4	0.4	0.4	0.4	0.4	0.5
3	0.8	0.7	0.9	0.3	0.7	0.8	0.4	0.4	0.3	0.4	0.3	0.4
4	0.9	0.7	0.9	0.4	0.7	0.7	0.3	0.3	0.3	0.4	0.4	0.4
5	0.7	0.7	0.7	0.4	0.7	0.7	0.4	0.3	0.3	0.3	0.5	0.5
6	0.7	0.6	0.7	0.4	0.6	0.7	0.5	0.4	0.3	0.4	0.3	0.4
7	0.8	0.7	0.8	0.5	0.7	0.8	0.4	0.3	0.3	0.5	0.4	0.5
8	0.8	0.6	0.8	0.4	0.7	0.9	0.4	0.2	0.2	0.3	0.3	0.4
9	0.9	0.7	0.8	0.3	0.6	0.9	0.5	0.3	0.2	0.3	0.4	0.5
10	0.9	0.8	0.8	0.3	0.7	0.9	0.5	0.2	0.3	0.3	0.4	0.4

Table 6: Binary fuzzy equivalence relation $\underset{\sim}{R}_C$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	0.72	0.80	0.37	0.64	0.79	0.41	0.31	0.29	0.37	0.38	0.44
2	0.72	1	0.72	0.37	0.64	0.72	0.41	0.31	0.29	0.37	0.38	0.44
3	0.80	0.72	1	0.37	0.64	0.79	0.41	0.31	0.29	0.37	0.38	0.44
4	0.37	0.37	0.37	1	0.37	0.37	0.37	0.31	0.29	0.37	0.37	0.37
5	0.64	0.64	0.64	0.37	1	0.64	0.41	0.31	0.29	0.37	0.38	0.44
6	0.79	0.72	0.79	0.37	0.64	1	0.41	0.31	0.29	0.37	0.38	0.44
7	0.41	0.41	0.41	0.37	0.41	0.41	1	0.31	0.29	0.37	0.38	0.41
8	0.31	0.31	0.31	0.31	0.31	0.31	0.31	1	0.29	0.31	0.31	0.31
9	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	1	0.29	0.29	0.29
10	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.31	0.29	1	0.37	0.37
11	0.38	0.38	0.38	0.37	0.38	0.38	0.38	0.31	0.29	0.37	1	0.38
12	0.44	0.44	0.44	0.37	0.44	0.44	0.41	0.31	0.29	0.37	0.38	1

The α-cut matrix $R_{C\alpha}$ is calculated by using eq. (6) in Table 6 and after calculating the eq. (7), the fuzzy equivalence classification is obtained by using α-cut.

In this paper, in agreement with the technicians and experts, the reasonable value of α is taken as 0.64, and 5 criteria such as the fitness of grinding product, content of impurities, capacity of the mill, the specific power consumption and reliability of the mill are selected as the main appraisal criteria with decisive influence.

3.1.2. Setting the weights of the main criteria by using FSFDMW

We determine the weight value of the importance degree of the five criteria for selecting type of mill by the subjective opinion data of ten decision makers (technicians and experts) according to the method discussed earlier.

The fuzzy value representation data for each criterion's importance degree of decision makers are presented in Table 7.

Table 7: Opinion data of decision makers for determining the weight value of the main criteria

	Fitness of grinding product	**Content of impurities**	**Capacity of mill**	**Specific power consumption**	**reliability of mill**
1	0.8	0.8	0.5	0.5	1
2	1	0.5	0.4	0.2	0.7
3	0.7	0.6	0.5	0.3	1
4	1	0.7	0.5	0.4	0.8
5	0.8	0.6	0.6	0.5	0.6
6	0.7	0.7	0.7	0.5	0.6
7	0.6	0.6	0.8	0.7	0.8
8	0.8	0.6	0.7	0.7	0.8
9	0.7	0.6	0.5	0.3	0.6
10	0.8	0.6	0.5	0.4	0.8

By using eq. (9), the normalized weight value of the decision makers for each criterion are calculated as shown in Table 8.

Table 8: Normalized weight value of the decision makers for each criterion

	Fitness of grinding product, x_1	Content of impurities, x_2	Capacity of mill, x_3	Specific power consumption, x_4	Reliability of mill, x_5
W_{j1}	0.0699665	0.0699665	0.0599713	0.0499761	0.0499761
W_{j2}	0.0600789	0.0600789	0.0700921	0.0600789	0.0500658
W_{j3}	0.0699671	0.0699671	0.0799624	0.0499765	0.0599718
W_{j4}	0.0701910	0.0601637	0.0701910	0.0601637	0.0501365
W_{j5}	0.0798269	0.0498918	0.0698485	0.0498918	0.0598702
W_{j6}	0.0699798	0.0699798	0.0599827	0.0699798	0.0499855
W_{j7}	0.0600214	0.0600214	0.0700250	0.0700250	0.0600214
W_{J8}	0.0798921	0.0699056	0.0599191	0.0499326	0.0499326
W_{j9}	0.0699649	0.0499749	0.0699649	0.0599699	0.0599699
W_{j10}	0.0600509	0.0600510	0.0700595	0.0600510	0.0500425

By using eq. (10) and eq. (11), the normalized weight value of the decision makers for whole criteria are calculated as follows:

$w_1 = 0.0712, w_2 = 0.0518, w_3 = 0.0610, w_4 = 0.0690, w_5 = 0.0673,$

$w_6 = 0.0636, w_7 = 0.0660, w_8 = 0.0724, w_9 = 0.0538, w_{10} = 0.0661$

By using eq. (11), eq. (12) and eq. (13), the fuzzy score function value and the normalized weight values of the criteria on H_5 are calculated as shown in Table 9.

Table 9: Fuzzy score function value and the normalized weight value of the criteria on H_5

	Fitness of grinding product, x_1	Content of impurities, x_2	Capacity of mill, x_3	Specific power consumption, x_4	Reliability of mill, x_5
$S_j(H_5)$	0.790101	0.630576	0.569929	0.450885	0.768656
W_{x_j}	0.246126	0.196432	0.17754	0.140456	0.239446

As shown in Table 9, the fuzzy weight value order of the criteria for selecting type of mill is as follows: fineness of the grinding product, reliability of mill, content of impurities contained in the grinding product, capacity of mill and specific power consumption.

3.1.3. Selecting type of grinding mill by using FSFDMW-TOPSIS

Dominance evaluation opinion data of ten decision makers for *j*-th criterion c_j for selecting type of mill are given as shown in Table 10-14.

The dominance evaluation opinion data of the decision makers for the *j*-th criterion c_j allows the decision makers to give a value of 1 if the dominance degree of the mill type is comparatively the most dominant, and a value of 0 if the dominance degree is not dominant, and to express it as a value between 0 and 1 according to the dominance degree.

Table 10: Opinion data of decision makers for estimating dominance to the fitness of grinding product(c_1)

	A	B	C	D	E	F	G	H
1	0.7	0.6	0.8	0.9	0.9	0.8	0.6	0.5
2	0.8	0.7	0.8	0.9	0.8	0.7	0.7	0.6
3	0.7	0.7	0.7	0.9	0.9	0.7	0.8	0.5
4	0.8	0.6	0.7	0.8	0.9	0.7	0.7	0.4
5	0.7	0.7	0.8	0.8	0.9	0.8	0.7	0.4
6	0.5	0.6	0.8	0.8	0.9	0.8	0.7	0.5
7	0.8	0.5	0.9	0.9	0.9	0.7	0.6	0.4
8	0.8	0.5	0.7	0.8	0.9	0.7	0.6	0.4
9	0.7	0.6	0.8	0.7	0.9	0.7	0.7	0.4
10	0.7	0.6	0.8	0.9	0.9	0.8	0.6	0.5

Table 11: Opinion data of decision makers for estimating dominance to content of impurities (c_2)

	A	B	C	D	E	F	G	H
1	0.3	0.7	0.4	0.6	0.3	0.3	0.7	0.8
2	0.4	0.8	0.5	0.7	0.3	0.3	0.6	0.9
3	0.3	0.7	0.5	0.5	0.4	0.4	0.7	0.8
4	0.2	0.7	0.6	0.4	0.3	0.3	0.7	0.9
5	0.3	0.8	0.4	0.5	0.3	0.2	0.6	0.9
6	0.3	0.6	0.3	0.6	0.4	0.2	0.5	0.8
7	0.4	0.7	0.2	0.7	0.2	0.3	0.7	0.8
8	0.3	0.7	0.3	0.5	0.2	0.3	0.8	0.8
9	0.4	0.8	0.4	0.5	0.3	0.4	0.8	0.7
10	0.3	0.7	0.3	0.6	0.2	0.3	0.7	0.8

Table 12: Opinion data of decision makers for estimating dominance to capacity of mill (c_3)

	A	B	C	D	E	F	G	H
1	0.9	0.9	0.8	0.7	0.6	0.7	0.9	0.6
2	0.9	0.8	0.9	0.5	0.7	0.8	0.8	0.7
3	0.8	0.8	0.8	0.5	0.6	0.6	0.8	0.7
4	0.9	0.8	0.7	0.6	0.5	0.7	0.9	0.6
5	0.9	0.7	0.8	0.5	0.6	0.7	0.8	0.7
6	0.8	0.7	0.8	0.5	0.6	0.8	0.9	0.7
7	0.7	0.7	0.7	0.6	0.5	0.7	0.8	0.6
8	0.8	0.7	0.9	0.6	0.6	0.6	0.7	0.7
9	0.8	0.8	0.8	0.5	0.6	0.7	0.8	0.8
10	0.9	0.6	0.8	0.7	0.6	0.7	0.9	0.6

Table 13: Opinion data of decision makers for estimating dominance to specific power consumption (c_4)

	A	B	C	D	E	F	G	H
1	0.4	0.5	0.8	0.7	0.6	0.7	0.9	0.8
2	0.3	0.6	0.7	0.6	0.5	0.6	0.8	0.7
3	0.3	0.6	0.8	0.7	0.6	0.6	0.8	0.6
4	0.3	0.7	0.7	0.7	0.6	0.5	0.8	0.7
5	0.2	0.7	0.6	0.6	0.5	0.7	0.8	0.6
6	0.3	0.6	0.7	0.5	0.4	0.6	0.9	0.7
7	0.3	0.6	0.8	0.5	0.3	0.7	0.8	0.6
8	0.3	0.7	0.7	0.7	0.2	0.7	0.8	0.7
9	0.4	0.5	0.7	0.5	0.2	0.6	0.9	0.6
10	0.2	0.7	0.7	0.7	0.3	0.6	0.8	0.7

Table 14: Opinion data of decision makers for estimating dominance to reliability of mill (c_5)

	A	B	C	D	E	F	G	H
1	0.9	0.7	0.9	0.8	0.6	0.8	0.9	0.8
2	0.8	0.7	0.8	0.8	0.5	0.7	0.8	0.7
3	0.7	0.7	0.8	0.7	0.6	0.8	0.8	0.8
4	0.8	0.8	0.8	0.7	0.6	0.8	0.8	0.8
5	0.9	0.8	0.9	0.8	0.7	0.8	0.9	0.7
6	0.9	0.7	0.9	0.8	0.7	0.9	0.9	0.8
7	0.8	0.8	0.9	0.8	0.7	0.8	0.8	0.7
8	0.9	0.8	0.9	0.8	0.6	0.8	0.9	0.8
9	0.9	0.7	0.9	0.8	0.7	0.9	0.9	0.7
10	0.9	0.7	0.9	0.8	0.6	0.8	0.8	0.8

By using eqs. (14)-(18), the fuzzy score function value matrix with the weights of decision makers is calculated as follows: (Table 15)

Table 15: Fuzzy score function value matrix with weights of decision makers

	Fitness of grinding product	Content of impurities	Capacity of mill	Specific power consumption	Reliability of mill
A	0.125061083	0.078391353	0.142283713	0.061799721	0.143884455
B	0.1070528	0.1765506	0.1267748	0.1279762	0.1250365
C	0.1356339	0.0950398	0.1356062	0.1481481	0.1471201
D	0.1459014	0.1370148	0.0969313	0.1276909	0.1317416
E	0.1572072	0.0707339	0.0998297	0.0850646	0.0650233
F	0.1305613	0.0736832	0.1185306	0.1297366	0.1370625
G	0.1197441	0.1676446	0.1407171	0.1710791	0.1420329
H	0.0788382	0.2009416	0.1393266	0.1485048	0.1080986

By using eq. (19), the decision making matrix with the fuzzy score function values and the weights of criteria is calculated as follows: (Table 16)

Table 16: Decision making matrix with the fuzzy score function values and the weights of criteria

	Fitness of grinding product	Content of impurities	Capacity of mill	Specific power consumption	Reliability of mill
A	0.031260662	0.015429177	0.02519052	0.008431992	0.034493347
B	0.0267593	0.0347491	0.0224448	0.0174612	0.0299749
C	0.0339035	0.018706	0.0240083	0.0202134	0.035269
D	0.03647	0.0269676	0.0171611	0.0174222	0.0315823
E	0.039296	0.013922	0.0176743	0.0116063	0.015588
F	0.0326355	0.0145025	0.0209852	0.0177013	0.0328579
G	0.0299316	0.0329962	0.0249132	0.0233421	0.0340495
H	0.0197066	0.0395498	0.024667	0.0202621	0.0259144

By using eq. (20) and eq. (21) in Table 16, the positive ideal solution (PIS) and the negative ideal solution (NIS) are calculated and then by using eq. (22) and eq. (23), the distance between the dominance evaluation value for the selection is calculated as follows: (Table 17 and Table 18)

Table 17: Distance between the dominance evaluation value for the positive ideal solution (PIS)

	A	B	C	D	E	F	G	H
D_t^+	0.029	0.016	0.022	0.017	0.035	0.027	0.011	0.022

Table 18: Distance between the dominance evaluation value for the negative ideal solution(NIS)

	A	B	C	D	E	F	G	H
D_t^-	0.024	0.028	0.028	0.028	0.020	0.024	0.033	0.031

By using eq. (24), the priority value of each selection is calculated as follows: (Table 19)

Table 19: Priority value of each selection

A	B	C	D	E	F	G	H
0.445	0.641	0.564	0.627	0.361	0.469	0.742	0.585

From Table 19, the most suitable type of mill for dental gypsum grinding is the impact mill with a priority value of 0.742.

3.2. Type selection of air classifier in dental gypsum grinding-classification process

One of the major technical requirements of the establishment of the dental gypsum grinding-classification process is to ensure the sufficient size distribution characteristics of the powder products required.

In general, in grinding-classification process requiring relatively high classification accuracy, all classifiers are precision classifiers, i.e., centrifugal air classifiers using a centrifugal force field as the classification force field.

In this paper, we choose O-Sepa type air classifier (a), ATP type air classifier (b), MS type air classifier (c) and MSS type air classifier (d) as the typical types of air classifiers applicable to dental gypsum grinding-classification process, and choose a suitable type of air classifier for the establishment of dental gypsum grinding-classification process using the fuzzy group decision making method.

3.2.1 Setting the main criteria for selecting type of air classifier by using FSFDMW

In this paper, we construct 10 technicians and experts as decision makers for selecting type of the air classifier, and then the opinion data of the decision makers for selecting the main criteria for the 11 possible criteria are listed in Table 20.

Table 20: Opinion data of the decision makers for selecting the main criteria of air classifier

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier	Manufacturing cost	Manufacturing Possibility	Easy of repair	suitability of operation	Life of classifier
1	0.8	0.8	0.8	0.8	0.7	0.9	0.4	0.3	0.4	0.4	0.4
2	0.9	0.8	0.9	0.7	0.6	0.8	0.3	0.3	0.3	0.4	0.5
3	0.8	0.7	0.9	0.8	0.7	0.8	0.4	0.3	0.4	0.3	0.4
4	0.7	0.7	0.7	0.7	0.7	0.8	0.5	0.3	0.3	0.4	0.4
5	0.9	0.7	0.9	0.8	0.7	0.8	0.3	0.3	0.4	0.5	0.5
6	0.7	0.6	0.7	0.6	0.6	0.7	0.4	0.3	0.3	0.4	0.4
7	0.9	0.7	0.8	0.8	0.7	0.9	0.5	0.2	0.4	0.5	0.5
8	0.8	0.6	0.8	0.7	0.7	0.9	0.2	0.2	0.3	0.3	0.4
9	0.8	0.7	0.8	0.8	0.8	0.9	0.3	0.3	0.3	0.4	0.5
10	0.9	0.8	0.8	0.8	0.7	0.9	0.2	0.3	0.3	0.4	0.4

The main criteria for selecting type of air classifier in the same way as for selecting type of mill are as follows: i.e. cut size, classification efficiency, classification accuracy, capacity, power consumption, and reliability.

3.2.2 Setting the weights of the main criteria by using FSFDMW

According to the subjective opinion data of 10 decision makers composed of technicians and experts, the weight of the importance degree of the six criteria for selecting type of air classifier is determined by the method of determining the weight value of criteria for selecting type of mill.

The fuzzy value representation data for determining the weight value of the importance degree to each criterion of decision makers are shown in Table 21.

Table 21: Opinion data of 10 decision makers for determining the weight value of criteria in air classifier

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
1	0.7	0.7	0.6	0.5	0.5	0.8
2	0.6	0.6	0.7	0.6	0.5	0.8
3	0.7	0.7	0.8	0.5	0.6	0.7
4	0.7	0.6	0.7	0.6	0.5	0.8
5	0.8	0.5	0.7	0.5	0.6	0.7
6	0.7	0.7	0.6	0.7	0.5	0.8
7	0.6	0.6	0.7	0.7	0.6	0.8
8	0.8	0.7	0.6	0.5	0.5	0.7
9	0.7	0.5	0.7	0.6	0.6	0.8
10	0.6	0.6	0.7	0.6	0.5	0.7

The fuzzy score function values and the normalized weight values of the criteria for determining the weight value of the criteria in classifier are as follows: (Table 22)

Table 22: Fuzzy score function values and the normalized weight values of the criteria

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
$S_j(H_6)$	0.068995772	0.0620513	0.0680252	0.0579487	0.0539496	0.0760504
W_{x_j}	0.17828	0.16021	0.17571	0.14988	0.13953	0.19639

3.2.3 Selecting type of air classifier by using FSFDMW-TOPSIS

Dominance evaluation opinion data of ten decision makers for j-th criterion c_j $(j = 1,2,3,4,5,6)$ for selecting type of air classifier are given as shown in Table 23-28.

Table 23: Opinion data of decision makers for estimating dominance to cut size (c_1)

	a	b	c	d
1	0.8	0.9	0.8	0.9
2	0.7	0.8	0.8	0.9
3	0.8	0.8	0.8	0.8
4	0.8	0.9	0.8	0.9
5	0.8	0.9	0.8	0.9
6	0.7	0.8	0.7	0.8
7	0.8	0.9	0.8	0.9
8	0.9	0.9	0.9	0.9
9	0.8	0.9	0.8	0.9
10	0.8	0.8	0.8	0.8

Table 24: Opinion data of decision makers for estimating dominance to classification efficiency (c_2)

	a	b	c	d
1	0.8	0.8	0.8	0.9
2	0.8	0.8	0.8	0.8
3	0.9	0.9	0.9	0.9
4	0.8	0.9	0.8	0.9
5	0.8	0.8	0.8	0.8
6	0.9	0.9	0.9	0.9
7	0.8	0.8	0.8	0.8
8	0.8	0.8	0.8	0.8
9	0.8	0.9	0.8	0.9
10	0.8	0.8	0.8	0.8

Table 25: Opinion data of decision makers for estimating dominance to classification accuracy (c_3)

	a	b	c	d
1	0.8	0.8	0.8	0.8
2	0.8	0.8	0.9	0.8
3	0.8	0.8	0.8	0.8
4	0.9	0.9	0.9	0.9
5	0.9	0.8	0.9	0.8
6	0.8	0.8	0.8	0.8
7	0.8	0.8	0.9	0.8
8	0.8	0.8	0.8	0.8
9	0.9	0.8	0.9	0.8
10	0.8	0.8	0.9	0.8

Table 26: Opinion data of decision makers for estimating dominance to capacity of classifier (c_4)

	a	b	c	d
1	0.7	0.6	0.8	0.7
2	0.8	0.7	0.8	0.7
3	0.7	0.6	0.8	0.6
4	0.7	0.6	0.7	0.6
5	0.7	0.7	0.8	0.7
6	0.8	0.8	0.8	0.8
7	0.7	0.6	0.8	0.6
8	0.8	0.7	0.8	0.7
9	0.7	0.6	0.7	0.6
10	0.7	0.7	0.8	0.7

Table 27: Opinion data of decision makers for estimating dominance to power consumption (c_5)

	a	b	c	d
1	0.5	0.6	0.8	0.6
2	0.6	0.7	0.7	0.5
3	0.7	0.7	0.9	0.7
4	0.6	0.8	0.8	0.7
5	0.7	0.7	0.8	0.6
6	0.7	0.67	0.9	0.7
7	0.6	0.7	0.9	0.7
8	0.7	0.8	0.9	0.7
9	0.7	0.7	0.8	0.7
10	0.6	0.7	0.8	0.7

Table 28: Opinion data of decision makers for estimating dominance to reliability of classifier (c_6)

	a	b	c	d
1	0.8	0.8	0.8	0.8
2	0.9	0.9	0.9	0.9
3	0.8	0.8	0.8	0.8
4	0.8	0.7	0.8	0.7
5	0.7	0.7	0.8	0.7
6	0.7	0.6	0.8	0.7
7	0.6	0.6	0.7	0.6
8	0.7	0.6	0.8	0.6
9	0.8	0.7	0.9	0.7
10	0.7	0.6	0.8	0.6

By using eq. (14)-(18), the fuzzy score function value matrix with the weights of decision makers is calculated as follows: (Table 29)

Table 29: Fuzzy score function value matrix with weights of decision makers

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
a	0.2376	0.2485	0.2505	0.2575	0.2253	0.2525
b	0.2590	0.2514	0.2449	0.2315	0.2499	0.2338
c	0.2410	0.2459	0.2597	0.2758	0.2928	0.2761
d	0.2624	0.2542	0.2449	0.2352	0.2321	0.2377

By using eq. (19), the decision making matrix with the fuzzy score function values and the weights of criteria is calculated as follows: (Table 30)

Table 30: Decision making matrix with the fuzzy score function values and the weights of criteria

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
a	0.04242	0.03981	0.04402	0.03855	0.03142	0.04958
b	0.04623	0.04028	0.04305	0.03465	0.03484	0.04592
c	0.04302	0.03940	0.04564	0.04128	0.04083	0.05422
d	0.04684	0.04072	0.04305	0.03520	0.03237	0.04668

By using eq. (20)-(23), the positive ideal solution (PIS) and the negative ideal solution (NIS) are calculated, and then the distance between the dominance evaluation value for the selection is calculated as follows: (Table 31 and Table 32)

Table 31: Distance between the dominance evaluation value for the positive ideal solution (PIS)

	a	b	c	d
D_t^+	0.01185	0.01249	0.00404	0.01312

Table 32: Distance between the dominance evaluation value for the negative ideal solution(NIS)

	a	b	c	d
D_t^+	0.10126	0.10068	0.10861	0.10085

By using eq. (24), the priority value of each selection is calculated as follows: (Table 33)

Table 33: Priority value of each selection

	a	b	c	d
B_t	0.89524	0.88966	0.96417	0.88490

From Table 33, the most suitable type of air classifier for dental gypsum grinding-classification process is MS type air classifier with a priority value of 0.96417. (Fig 1)

Fig 1: Dental gypsum grinding-classification process

Conclusion

FSFDMW-TOPSIS is the method of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) using the fuzzy score function of decision makers' weights (FSFDMW).

In this paper, we selected the suitable types of mill and air classifier for the dental gypsum grinding-classification process by using FSFDMW-TOPSIS method based on the opinion data of technicians and experts.

The impact mill was selected as the most suitable type of mill with a value of 0.742 among the eight dry mills with five criteria and MS type air classifier was selected as the most suitable type of air classifier with a value of 0.96417 among the four air classifiers with six criteria.

Acknowledgements

The authors are grateful to teachers of Applied Mathematics Faculty in Kim Cheak University of Technology for their assistance to accomplish this work.

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