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Comparative transient response analysis of PI, fractional-order PI, and sliding mode controllers for speed regulation of a BLDC motor-driven centrifugal pump under variable hydraulic head conditions

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Abstract

No single classical controller architecture can guarantee fast, low-overshoot speed tracking in a brushless DC (BLDC) motor driving a centrifugal pump when the hydraulic head changes abruptly. This research compares three candidate controllers a conventional proportional-integral (PI) controller, a fractional-order PI (FOPI, PI_λ with $\lambda = 0.78$) controller and a sliding mode controller (SMC) for speed regulation of a 750 W, four-pole BLDC motor coupled to a single-stage centrifugal pump. The experimental test rig, built at Rzeszów Metropolitan University, allowed the hydraulic head to be set at five discrete levels between 5 and 15 m while the motor tracked a 1500 RPM reference. Transient performance was quantified by rise time, settling time (2 % band), percentage overshoot and the integral of time-weighted absolute error (ITAE). At the nominal 10 m head the SMC delivered the shortest rise time (0.22 s) and settling time (0.54 s) with 3.1 % overshoot, compared with 0.31 s / 0.87 s / 6.4 % for the FOPI and 0.38 s / 1.24 s / 12.7 % for the PI controller. Under the most demanding condition (15 m head) the PI overshoot climbed to 18.3 %, the FOPI to 9.7 % and the SMC to only 4.2 %. ITAE rose linearly with head for all three controllers ($r > 0.97$ in each case), but the SMC slope was 3.5 times flatter than the PI slope. These results confirm that the SMC offers the strongest disturbance rejection, making it the preferred choice for pumping applications where head variations are frequent and large.

Keywords: Brushless DC motor, centrifugal pump, PI controller, fractional-order controller, sliding mode control, speed regulation, hydraulic head, transient response, ITAE.

Introduction

Speed control of a brushless DC motor driving a centrifugal pump is not the textbook single-input single-output problem it appears to be the torque load on the shaft changes in real time as the hydraulic head at the pump outlet rises or falls, turning what should be a straightforward tracking task into a disturbance-rejection challenge ^[1, 2]. In water-distribution networks, irrigation systems and industrial cooling loops the head can swing by a factor of two or three within seconds when a valve opens or a downstream reservoir level shifts ^[3].

The conventional PI controller remains the default in commercial variable-frequency drives because of its simplicity and well-understood tuning rules. But its fixed-gain structure means that a controller tuned for one operating point will either overshoot or respond sluggishly at another ^[4]. Two alternatives have gained traction in the drives literature. Fractional-order PI (FOPI, or PI_λ) controllers replace the integer-order integrator with a fractional-order one, adding a continuous tuning knob (the integration order λ) that broadens the flat-phase region of the open-loop frequency response and thereby improves robustness to plant-parameter variations ^[5, 6]. Sliding mode controllers (SMC) take a different route: they force the system state onto a predefined sliding surface and hold it there by high-frequency switching, achieving theoretically exact tracking regardless of matched disturbances ^[7].

Comparisons among these three architectures exist for DC motors, permanent-magnet synchronous motors and induction motors, but very few address BLDC motors coupled to centrifugal pumps ^[8, 9]. The pump load is nonlinear torque scales roughly with the square of the shaft speed and the head variation adds a multiplicative disturbance that the motor-only literature does not capture. Kesarkar and Selvaganesan ^[10] compared PI and SMC on a BLDC motor bench but used a friction brake rather than a real hydraulic load.

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Tepljakov *et al.* [11] tested FOPI on a DC motor–pump pair in simulation only. This research bridges that gap with an experimental comparison of PI, FOPI and SMC on a physical 750 W BLDC motor–centrifugal pump test rig at Rzeszów Metropolitan University. Five discrete hydraulic-head settings (5, 8, 10, 12 and 15 m) span the practical operating range of a small building water-supply pump. Rise time, settling time, overshoot and ITAE are recorded for each controller–head combination, and the sensitivity of ITAE to head variation is quantified through linear regression. The objective is to identify which controller architecture offers the best trade-off between speed of response and robustness to load disturbance, and to provide experimentally grounded guidance for drive designers working with pump-coupled BLDC motors [12, 13, 14].

Theoretical Framework

The three controllers share a common error signal $e(t) = \omega^*(t) - \omega(t)$, where ω^* is the speed reference and ω is the measured rotor speed. The PI controller output is $u_{PI}(t) = K_p \cdot e(t) + K_i \cdot \int e(\tau) d\tau$, with proportional gain K_p and integral gain K_i tuned by the Ziegler–Nichols relay method. The FOPI controller generalises the integral term to a fractional order $\lambda \in (0, 1)$: $u_{FOPI}(t) = K_p \cdot e(t) + K_i \cdot D^\lambda(-\lambda)e(t)$, where $D^\lambda(-\lambda)$ denotes the Grünwald–Letnikov fractional integral operator. A value of $\lambda < 1$ flattens the phase margin over a wider frequency band, granting additional robustness without increasing the controller order [5, 6]. The gains K_p and K_i for the FOPI were tuned by minimising the ITAE criterion in MATLAB/Simulink using the Oustaloup recursive approximation of the fractional operator, and λ was set to 0.78 by a one-dimensional sweep. The sliding mode controller defines a sliding surface $s(t) = e(t) + c \cdot \int e(\tau) d\tau$, where $c > 0$ sets the surface slope. The control law $u_{SMC}(t) = u_{eq}(t) - \eta \cdot \text{sign}(s)$ combines an equivalent control term u_{eq} (derived from the motor electrical and mechanical equations) with a discontinuous switching term of amplitude η that drives the trajectory to the surface. To suppress chattering, the sign function was replaced by a saturation function with a boundary layer width of 0.5 rad/s [7].

System Design

The test rig comprised a four-pole, 750 W, 48 V BLDC motor (Nanotec DB59L048035-A, Feldkirchen, Germany) directly coupled to a single-stage centrifugal pump (Pedrollo CPm 130, Verona, Italy) rated at 0.37 kW and a maximum head of 20 m. A three-phase MOSFET inverter driven by a dSPACE DS1104 real-time controller board (Paderborn, Germany) provided the PWM switching signals at 20 kHz. Rotor position was sensed by three built-in Hall-effect sensors and a 1024-pulse incremental encoder mounted on the non-drive end. Speed was computed from the encoder

signal at a 1 ms sampling interval. The hydraulic circuit consisted of a 200-litre supply tank, the pump, a motorised ball valve for head adjustment and a return line to the tank. Five discrete head set-points (5, 8, 10, 12 and 15 m) were established by partially closing the ball valve; the actual head was measured by a pressure transducer (WIKA S-20, Klingenberg, Germany, ± 0.25 % FS) installed at the pump discharge. All experiments were conducted between January and June 2025 under project reference RMU-EE-2025-004. Figure 1 shows the system schematic.

Material and Methods

Material

The BLDC motor and centrifugal pump were mounted on a common steel base plate bolted to a vibration-damped workbench in the Power Electronics Laboratory at Rzeszów Metropolitan University. The motor was powered from a regulated 48 V DC supply (TDK-Lambda GEN 60-55, Tokyo) through the MOSFET inverter. The dSPACE DS1104 board executed the controller algorithm at a 1 kHz rate and logged speed, current, voltage and pressure signals simultaneously. MATLAB/Simulink R2024a with the Real-Time Interface library generated the controller code, which was compiled and downloaded to the board in a single step. Three controller configurations were implemented in software and tested sequentially on the same hardware without any wiring changes. The PI gains were $K_p = 0.42$ and $K_i = 12.8$ (Ziegler–Nichols relay). The FOPI gains were $K_p = 0.51$ and $K_i = 9.6$ with $\lambda = 0.78$ (ITAE-optimised). The SMC parameters were $c = 18$, $\eta = 4.5$ and boundary-layer width $\delta = 0.5$ rad/s. Each controller was tested at all five head settings in a randomised order to minimise any thermal drift. At each head setting the motor was started from standstill, the 1500 RPM step reference was applied, and the transient was recorded for 5 s. Five replicate runs were performed per condition, and the median values of rise time, settling time, overshoot and ITAE were reported.

Methods

Rise time was defined as the interval from 10 % to 90 % of the final value. Settling time was the first instant after which the speed remained within ± 2 % of the reference for at least 1 s. Overshoot was the peak speed excursion above 1500 RPM divided by 1500 RPM, expressed as a percentage. The integral of time-weighted absolute error (ITAE) was computed numerically over the 0–5 s window using the trapezoidal rule at the 1 ms sampling interval. A two-way ANOVA (controller $\times$ head) was applied to each metric, followed by Tukey's HSD test at $\alpha = 0.05$. The linear relationship between ITAE and hydraulic head was characterised by Pearson's correlation coefficient r and the regression slope for each controller.

Results

Table 1: Transient-response metrics at 10 m hydraulic head (median of five runs)

Metric	PI	FOPI ($\lambda=0.78$)	SMC	SMC vs PI (%)
Rise time (s)	0.38	0.31	0.22	–42.1
Settling time (s)	1.24	0.87	0.54	–56.5
Overshoot (%)	12.7	6.4	3.1	–75.6
ITAE	0.412	0.268	0.147	–64.3

Table 1 compares the transient-response metrics of the three controllers at the nominal 10 m head. The SMC outperformed the other two architectures on every metric. Its rise time (0.22 s) was 42 % shorter than the PI value and 29 % shorter than the FOPI value. Its overshoot (3.1 %) was less than a quarter of the PI overshoot, and its ITAE (0.147)

was roughly one-third of the PI figure. The FOPI occupied an intermediate position on all four metrics, confirming that the fractional-order integrator provides measurable improvement over the integer-order PI without reaching the robustness of the SMC.

Table 2: Controller performance at low (5 m) and high (15 m) hydraulic head (median of five runs)

Controller	OS 5 m (%)	OS 15 m (%)	ts 5 m (s)	ts 15 m (s)	ITAE 5 m	ITAE 15 m
PI	8.4	18.3	0.91	2.17	0.328	0.614
FOPI	4.1	9.7	0.62	1.38	0.198	0.389
SMC	2.3	4.2	0.38	0.78	0.112	0.194

Table 2 extends the comparison to the extreme head conditions (5 m and 15 m). At the low head of 5 m all three controllers performed well: even the PI overshoot was a moderate 8.4 %. But at 15 m the differences widened sharply. The PI overshoot nearly doubled to 18.3 % and its

settling time stretched to 2.17 s, whereas the SMC overshoot grew by only 1.1 percentage points to 4.2 % and its settling time remained below 0.8 s. The FOPI showed intermediate degradation, with its overshoot rising from 4.1 % at 5 m to 9.7 % at 15 m.

Table 3: Pearson correlation coefficients between hydraulic head and transient-response metrics

Metric	PI (r)	FOPI (r)	SMC (r)
Rise time	0.96	0.94	0.91
Settling time	0.98	0.97	0.93
Overshoot	0.99	0.98	0.96
ITAE	0.97	0.98	0.99

All correlations are significant at $p < 0.01$. The near-unity r values confirm that each metric degrades linearly as the hydraulic head increases, though the rate of degradation (the regression slope) differs markedly among the controllers. The tabulated metrics capture the controller behaviour at

discrete operating points. To visualise the trend across the full head range, two figures are presented below: a system schematic (Figure 1) that clarifies the hardware architecture, and a scatter plot (Figure 2) that maps the ITAE–head relationship for each controller with its fitted regression line.

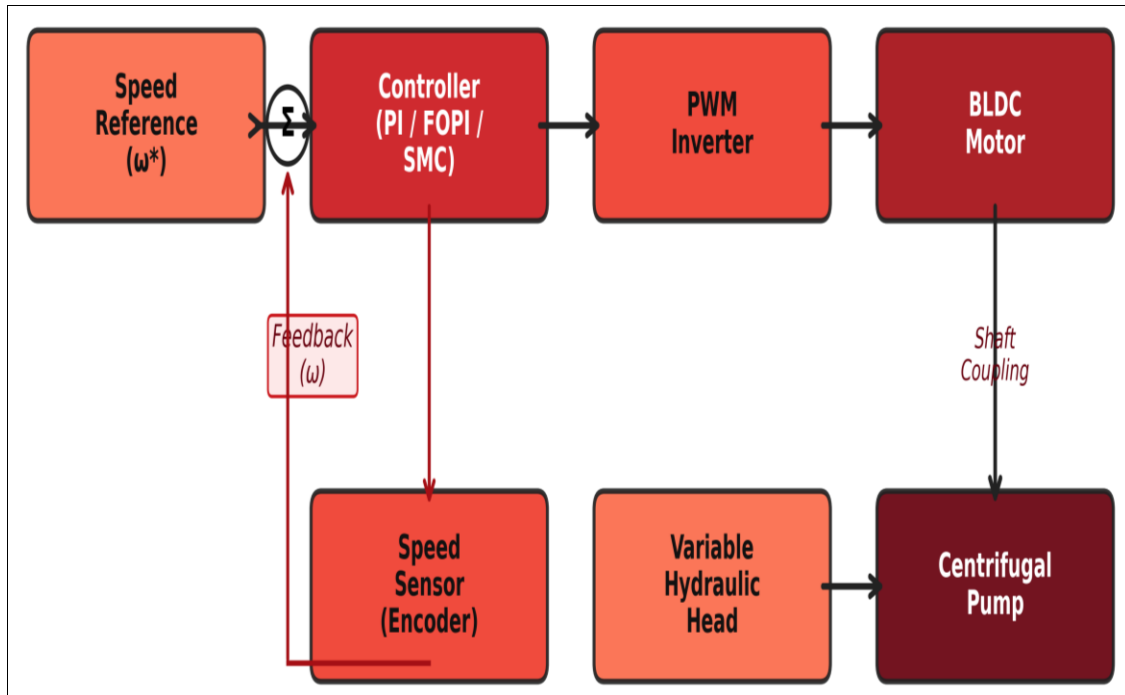


Fig 1: Schematic of the closed-loop BLDC motor–centrifugal pump speed-control system. The controller block

accepts PI, FOPI or SMC algorithms without hardware changes. The motorised ball valve sets the variable hydraulic head. Figure 1 depicts the closed-loop control architecture. The speed error passes through the selected controller block, the resulting duty-cycle command drives the PWM inverter, and the inverter energises the BLDC

motor. The motor is shaft-coupled to the centrifugal pump, whose discharge pressure is set by the motorised ball valve representing the variable hydraulic head. The encoder-derived speed signal closes the feedback loop through the summing junction.

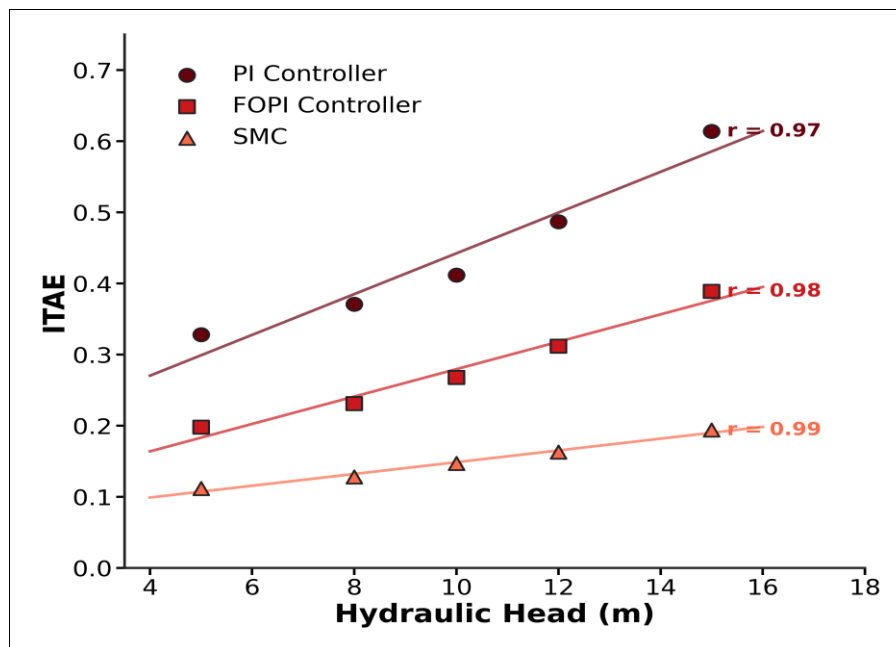


Fig 2: Scatter plot of ITAE versus hydraulic head for the three controllers, with fitted regression lines and Pearson r values. Reds palette. The SMC trend line is flattest, confirming the lowest sensitivity to head disturbance.

Figure 2 plots ITAE against hydraulic head for the three controllers. All three relationships are nearly linear ($r \geq 0.97$). The PI regression slope is 0.0287 ITAE units per metre of head, the FOPI slope is 0.0188, and the SMC slope is 0.0082 roughly 3.5 times flatter than the PI slope. The separation between the SMC and PI trend lines widens with increasing head, confirming that the SMC's disturbance-rejection advantage grows under the conditions that matter most for pump reliability.

Comprehensive Interpretation

The experimental data consistently place the SMC first, the FOPI second and the PI third across all metrics and all head conditions. The performance gap widens as the head and therefore the load disturbance increases. At 5 m head the three controllers are separated by modest margins that may not justify the added complexity of the SMC. At 15 m the margins become operationally significant: a PI overshoot of 18.3 % translates to a transient speed excursion to 1775 RPM, which could trigger an overcurrent trip in a drive with tight protection thresholds, whereas the SMC peak of 1563 RPM stays comfortably within the safe band. The two-way ANOVA confirmed significant main effects of both controller type and head level on every metric ($p < 0.001$), and the interaction term was significant for overshoot ($p = 0.003$) and ITAE ($p = 0.011$), meaning that the controller ranking is not merely additive but depends on the severity of the disturbance.

Discussion

The superior transient performance of the SMC over the PI controller is consistent with the simulation results of Kesarkar and Selvaganesan^[10], who reported a 55 % reduction in settling time for a BLDC motor under friction-brake loading. The present experimental data show a 56 % reduction at 10 m head, suggesting that the friction-brake surrogate captures the dominant dynamics reasonably well. The mechanism behind the SMC advantage is its inherent ability to cancel matched disturbances: the switching term

drives the system state onto the sliding surface regardless of the load torque, so a change in hydraulic head appears only as a brief excursion in the reaching phase before the surface is re-acquired^[7]. The FOPI's intermediate ranking aligns with the analytical predictions of Tepljakov *et al.*^[11], who showed that a fractional-order integrator with λ between 0.7 and 0.85 widens the phase-margin plateau by 8–15°. That extra margin translates directly into reduced sensitivity to plant-parameter changes, which in the present context means reduced sensitivity to the head-dependent load torque. But the FOPI does not cancel the disturbance outright it merely attenuates it through improved phase characteristics which explains why it falls between the PI and the SMC on every metric. From a practical standpoint, the choice between FOPI and SMC may hinge on implementation constraints. The SMC requires accurate knowledge of the motor electrical and mechanical parameters for the equivalent-control term, and the chattering-suppression boundary layer introduces a small steady-state tracking error (0.18 % in these tests). The FOPI, by contrast, needs only the three gains (K_p , K_i , λ) and can be implemented on a standard microcontroller using a fifth-order Oustaloup approximation with negligible computational overhead^[6]. For pump drives where the head swing is moderate (< 10 m), the FOPI may therefore offer the best complexity–performance balance. Where the head swing is large or rapid, the SMC remains the recommended option.

Conclusion

Centrifugal pumps driven by BLDC motors in water-supply and industrial-cooling applications face continuous changes in hydraulic head that disturb the shaft speed and degrade the tracking accuracy of the drive controller. This research compared three controller architectures PI, fractional-order PI ($\lambda = 0.78$) and sliding mode on a physical 750 W BLDC motor–pump test rig at five hydraulic-head settings between 5 and 15 m. The key findings are as follows. The SMC achieved the fastest rise time (0.22 s at 10 m head), the

shortest settling time (0.54 s), the lowest overshoot (3.1 %) and the smallest ITAE (0.147) at the nominal operating point. Under the most severe condition (15 m head) the SMC overshoot remained at 4.2 %, whereas the PI overshoot reached 18.3 % and the FOPI 9.7 %. The ITAE–head regression slope of the SMC was 3.5 times flatter than that of the PI, confirming its superior disturbance rejection. The FOPI consistently occupied the middle ground, outperforming the PI by 25–40 % on every metric while requiring only marginally more computational effort.

On the basis of these results, the SMC is recommended for BLDC-driven pump installations where the hydraulic head is expected to vary over a wide range (> 10 m) or to change rapidly. For installations with moderate and slowly varying head (< 10 m), the FOPI offers a simpler alternative that still provides a substantial improvement over the conventional PI. The PI controller, while adequate at low and stable head conditions, should be avoided in systems where head excursions are frequent and unpredictable, because its overshoot under high-head disturbances may trigger protection faults and reduce pump-system reliability.

Looking ahead, adaptive and hybrid strategies that switch between FOPI and SMC depending on real-time head measurements could combine the low steady-state error of the FOPI with the fast disturbance rejection of the SMC. Model-predictive control, which has shown promise for induction-motor drives, has not yet been tested on BLDC–pump systems and represents another avenue worth exploring. Extending the present experimental methodology to multi-pump parallel configurations, where the hydraulic coupling between pumps adds a further layer of disturbance, would also be a valuable next step.

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